

## ***The Extensive Form***

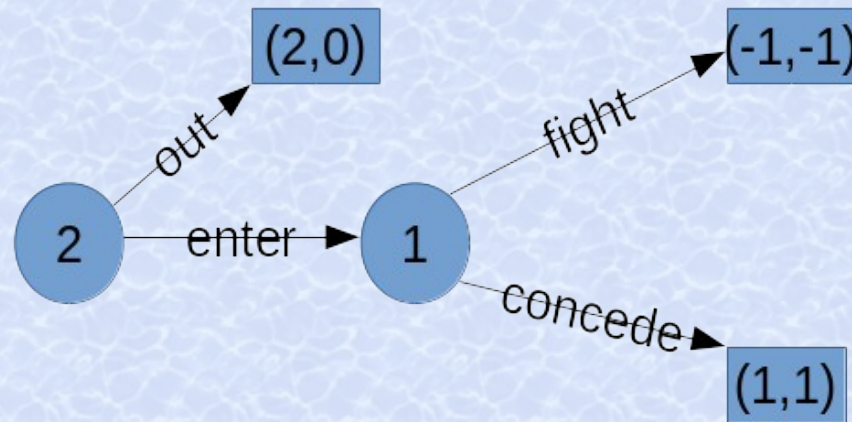
the *extensive form* of a game is a systematic way of laying out how players move over time and what they know when they move

best to illustrate this by an example

we will start with the case where players move sequentially and observe each other's play

(like chess)

## The Chain Store Game: Game Tree



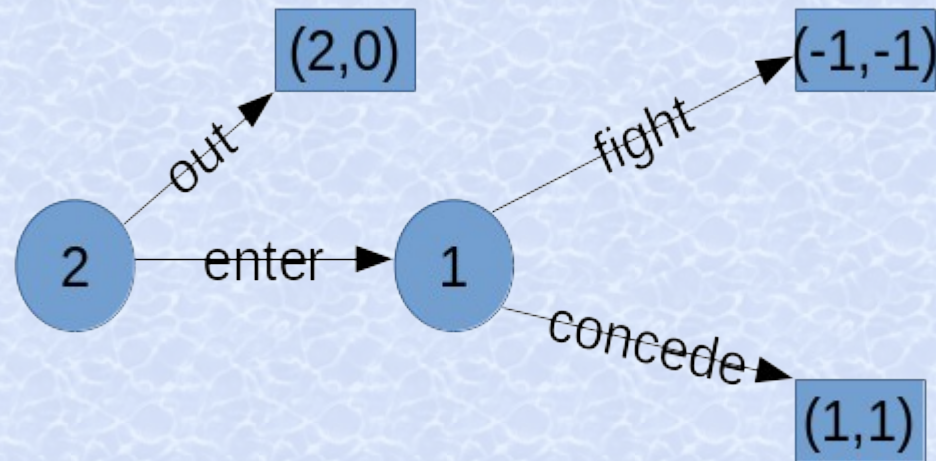
round nodes are *decision nodes*, the number indicates the player

the arrows are the *actions* or *decisions*

the square nodes are the *payoffs* to player 1 and player 2 respectively

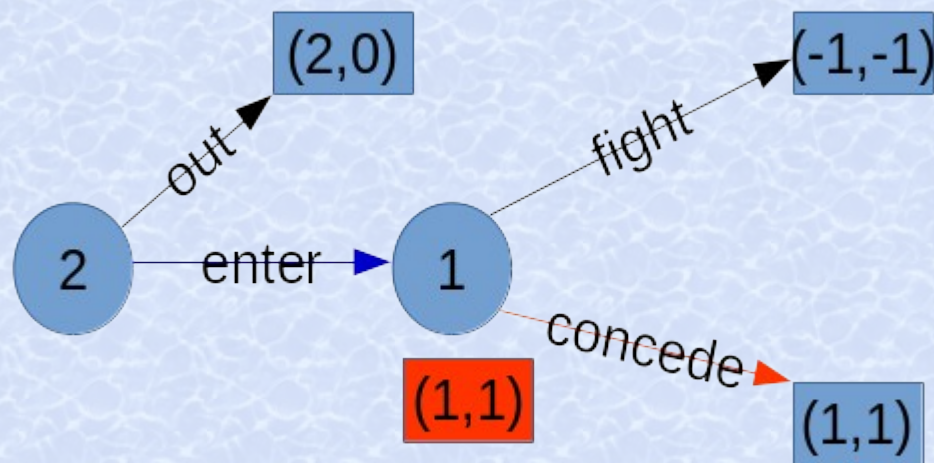
the game begins with the *root node*

## *The Chain Store Story*



Player 1 is an incumbent firm, player 2 a potential entrant

## Subgame Perfect Equilibrium



backward induction: start at the end to see what happens in the final move, then keep moving earlier in time

## Strategies

Give instructions to a friend or robot who will play on your behalf at each decision node, tell them what to do if they are there

player 1 has one decision node: two possible instructions

**fight concede**

player 1 has one decision node: two possible instructions

**enter out**

*normal form*

	<b>enter</b>	<b>out</b>
<b>fight</b>	-1,-1	2,0
<b>concede</b>	1,1	2,0

## *Nash Equilibrium*

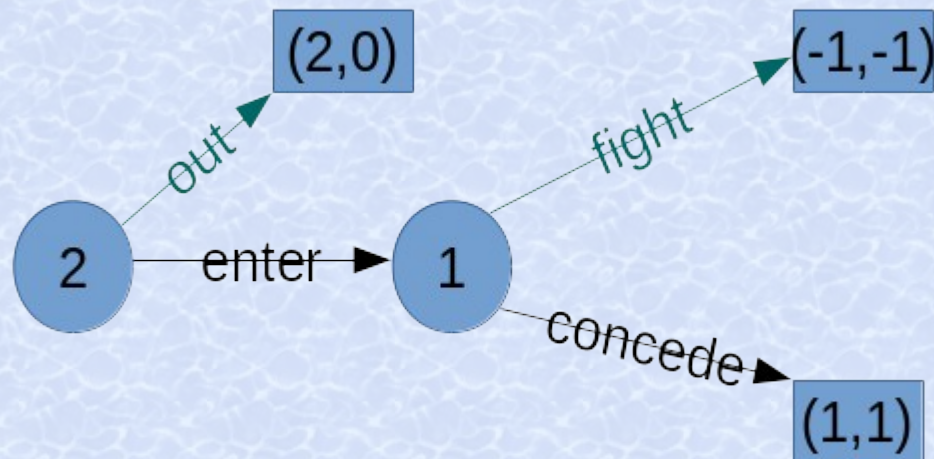
	<b>enter</b>	<b>out</b>
<b>fight</b>	-1,-1	2*,0*
<b>concede</b>	1*,1*	2*,0

two Nash equilibria:

concede enter (subgame perfect)

fight out

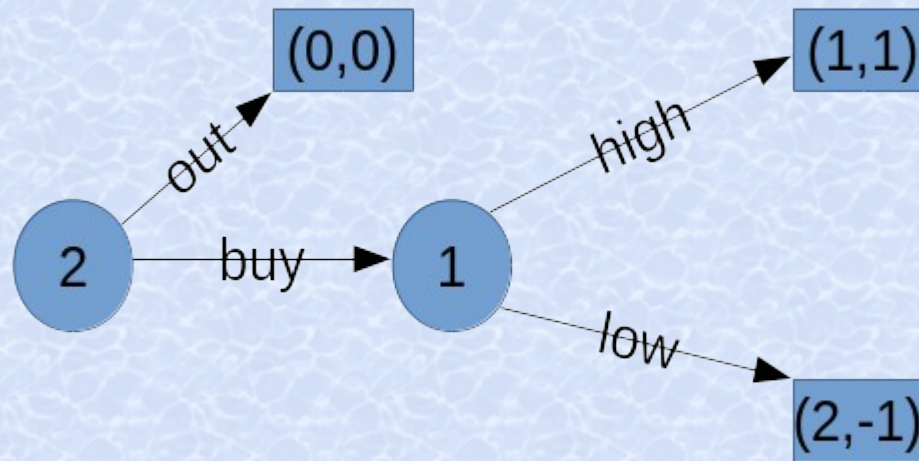
## *Threat of Fighting is not Credible*



importance of commitment or lack of commitment

## The Quality Game

Quality  
Game



player 1 is a firm that decides whether to provide a high or low quality product to player 2, the customer

subgame perfect: out-low

would like to commit to high quality

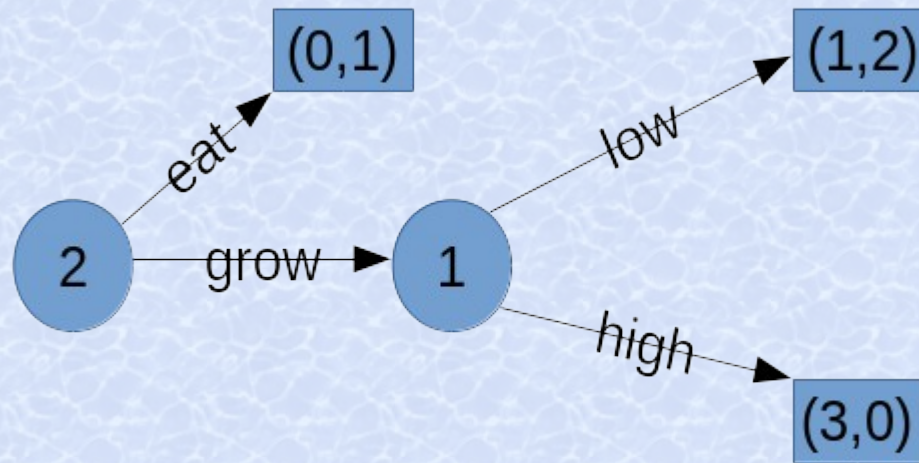


## *Normal Form of the Quality Game*

	<b>out</b>	<b>buy</b>
<b>high</b>	$0^*, 0$	$1, 1^*$
<b>low</b>	$0^*, 0^*$	$2^*, -1$

Only one Nash equilibrium (the subgame perfect one)

## Peasant Dictator Game



Player 2 is a peasant with a unit of seed corn that can be eaten

Or grown into three units of corn

But corn in the field can be taxed by the dictator, player 1

## *Political Economy Applications*

- time consistency of government policy
- capital taxation
- inflation



## *Stackelberg Equilibrium*

	4	5	8
4	32,32	28,35*	16*,32
5	35*,28	30*,30*	15,24
8	32*,16	24,15	0,0

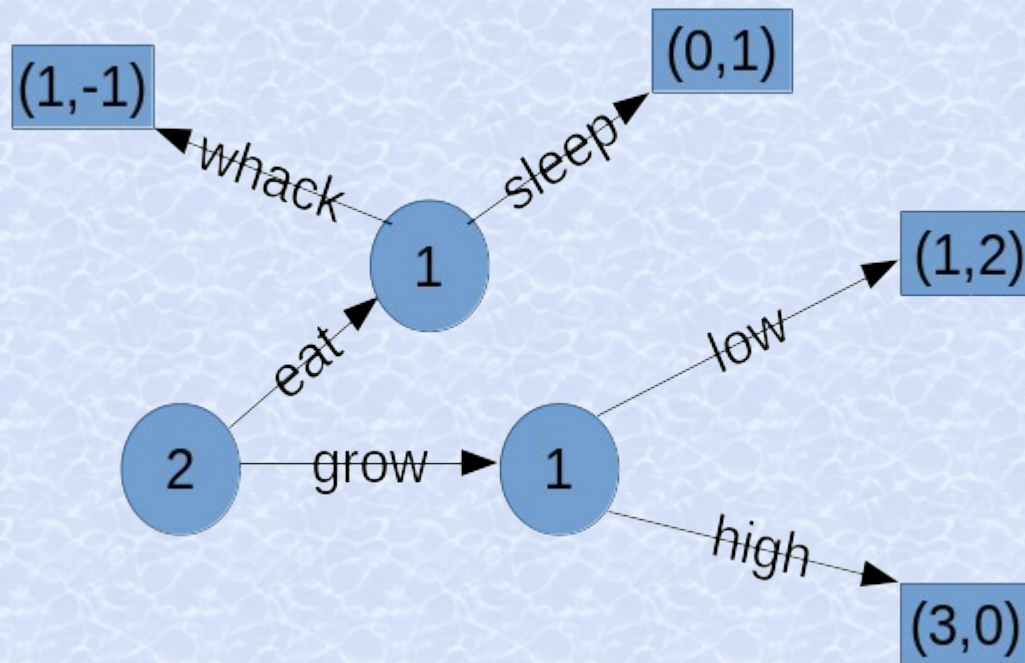
suppose that firm 1 can commit to a level of output and this is observed by firm 2 before firm 2 chooses their output

commit to 4? best response 5, get 28

commit to 5? best response 5, get 30

commit to 8? best response 4, get 32

## More About Strategies



How many strategies does player 1 have?

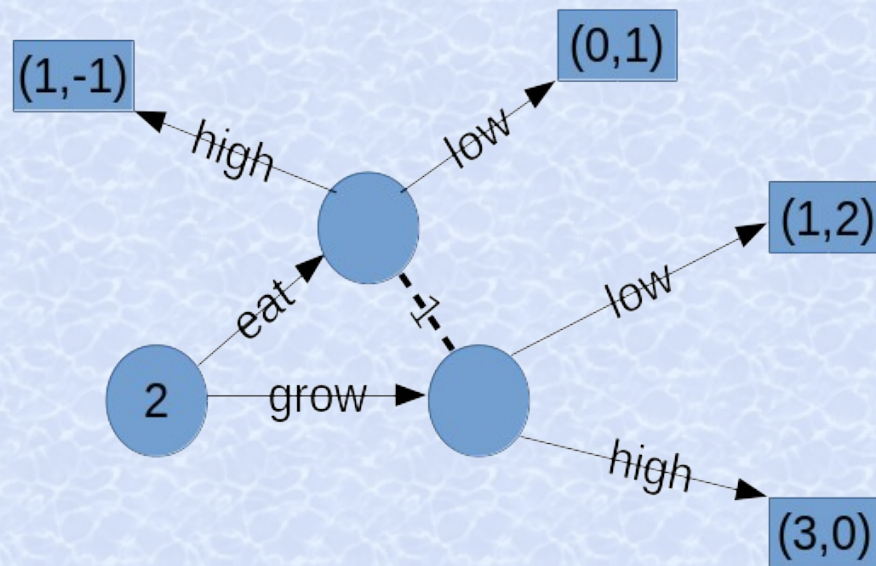
## *Normal Form of the Whacking Game*

	<b>eat</b>	<b>grow</b>
<b>sleep-high</b>	0,1*	3*,0
<b>sleep-low</b>	0,1	1,2*
<b>whack-high</b>	1*,-1	3*,0*
<b>whack-low</b>	1*,-1	1,2*

## Information Sets

How can we represent a simultaneous move game as an extensive form?

Not literally simultaneous, just ignorant of the other person's choice



Note conformal labelling of actions at an information set

## ***What is are Information Sets and Strategies?***

An information set connects several nodes for the same player  
the player does not know which node they are at, only that they are at the information set

a strategy assigns actions to information sets (conformal labelling)

normal form of the simultaneous peasant whacking game

	<b>eat</b>	<b>grow</b>
<b>high</b>	1*, -1	3*, 0*
<b>low</b>	0, 1	1, 2*



## ***Subgame Perfection in the Finitely Repeated PD***

		Firm 2	
		<b>4</b>	<b>5</b>
Firm 1			
<b>4</b>		32,32	28,35
<b>5</b>		35,28	30,30

Play this  $T$  times, everyone sees what happened last period  
get the sum of payoffs from each period

solve for subgame perfection by backwards induction

- What happens the last time the game is played?
- What happens the next to last time the game is played?
- What does this mean?

## Concepts

- **extensive form game**
- game tree, decision node, action, payoff node, root node
- backward induction, **subgame perfect equilibrium**
- **chain store game**, quality game, peasant-dictator game, Stackelberg game
- commitment, credibility
- **information set, strategy**
- **normal form**
- finitely repeated Prisoner's Dilemma

## ***Skill***

given a description of a game

draw the extensive form, find the subgame perfect equilibrium

count the strategies, find the strategies, find the normal form