Coordination Games

Airplane hijacking game

	attack	**wait**
attack	2*,2	0,0
wait	0,0	1*,1

no strategies are dominated: beliefs matter

example of a *coordination game*

Nash Equilibrium

each player plays optimally and correctly guesses what the other player will do

step 1: *best response* what is best to do given beliefs

step 2: equilibrium of beliefs

	attack	**wait**
attack	**2*,2***	0,0
wait	0,0	**1*,1***

Two Nash equilibria: which one?

Pareto ranked, one is “obvious”

Dominant Strategy versus Nash

Players playing dominant strategies is an example of Nash equilibrium

here beliefs do not matter

	Player 2	
Player 1	**don't confess**	**confess**
don't confess	32,32	28,35*
confess	35*,28	**30*,30***

Other Coordination Games

Drive on the left or on the right?

	left	**right**
left	**1*,1***	0,0
right	0,0	**1*,1***

Battle of the sexes

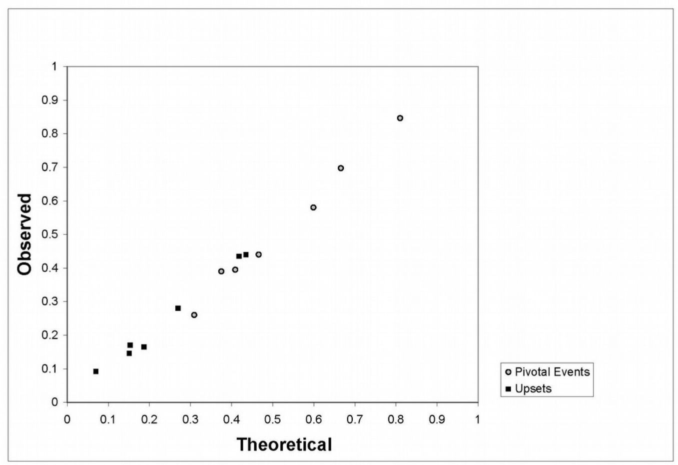
	opera	**match**
opera	**2*,1***	0,0
match	0,0	**1*,2***

Why Nash Equilibrium?

- reasoning versus learning
- at a Nash equilibrium, there is nothing further to learn
- the rush hour traffic game

Learning and Nash Equilibrium

- economists think people are pretty smart
- they are pretty good at learning
- algorithms take ages to converge
- people are quick



Pre-911 Airplane Hijacking Game

	attack	**wait**
attack	**1*,1***	0,0
wait	0,0	**2*,2***

Versus post 911 game

	attack	**wait**
attack	**2*,2***	0,0
wait	0,0	**1*,1***

Case Study: 911

1990s about 18 aircraft hijackings a year

most ended peacefully and the passengers never attacked

after 911 this dropped to just a few aircraft hijacking a year

most ended when the passengers attacked the hijackers

how long did it take to switch from one equilibrium to the other?

one hour and eleven minutes

Duopoly Again

profits

$$\pi_i = [16 - (Q_i + Q_{-i})]Q_i$$

note use of $-i$ to mean "the other player"

the *best response* or *reaction function* for player i maximizes their profit with respect to their own output Q_i based on their belief about their opponent output Q_{-i}

The Best Response

to do this take the partial derivative with respect to Q_i, set it equal to zero and solve for Q_i

$$\partial\pi_i/\partial Q_i = 16 - 2Q_i - Q_{-i} = 0$$

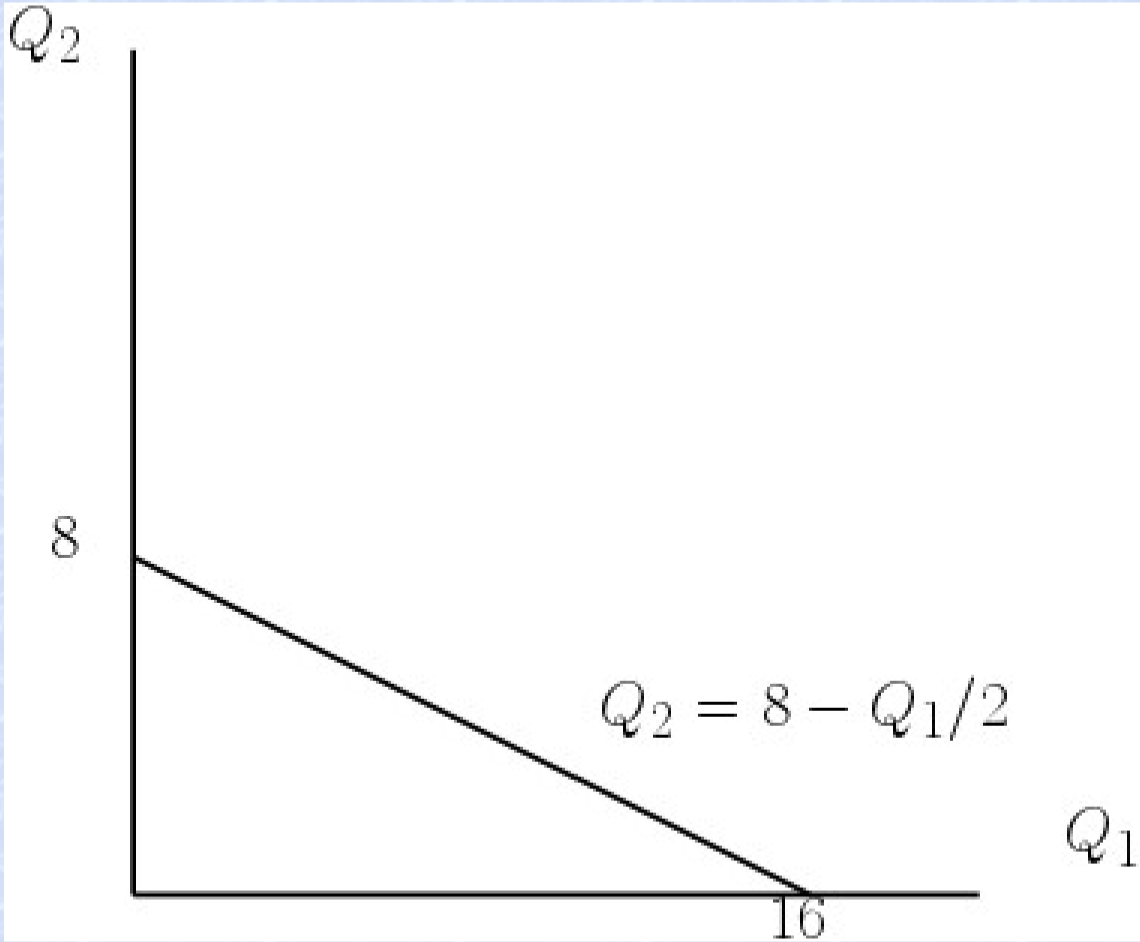
solution is the best response or reaction function

$$Q_i = 8 - \frac{Q_{-i}}{2}$$

equilibrium is where both player's beliefs are correct

that is to say: both are playing a best response at the same time

Best Response Graph



Equilibrium

Solve

$$Q_2 = 8 - Q_1/2, Q_1 = 8 - Q_2/2$$

solution

$$Q_1 = Q_2 = 16/3 = 5\,1/3$$

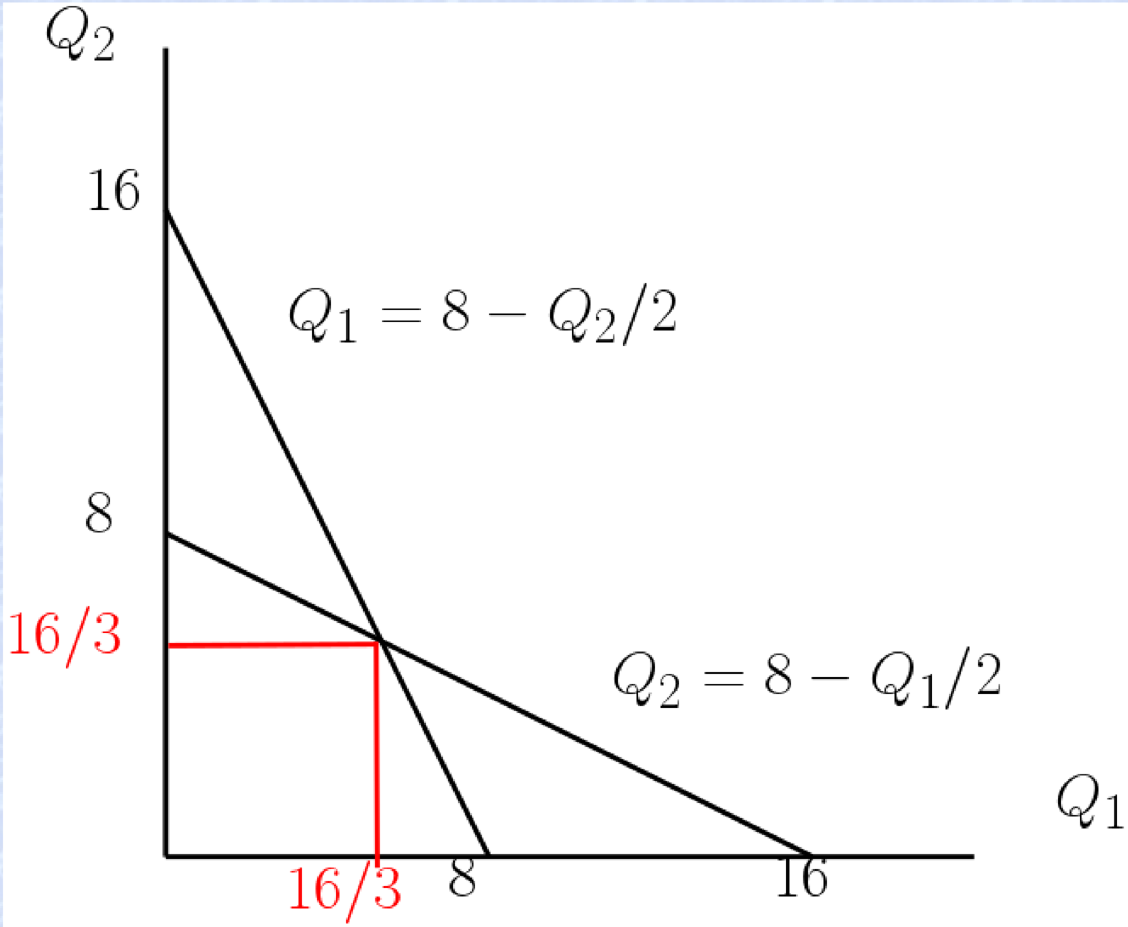
less than monopoly (8) but more than half monopoly

industry output

$$Q = Q_1 + Q_2 = 32/3 = 10\,2/3$$

more than monopoly but 2/3 of competitive (16)

Equilibrium : Graph



The Cournot Model

- an *oligopoly* market with n identical firms facing constant marginal cost *c*
- demand given by $p = a - bQ$

so that the competitive solution is $(a-c)/b$ units of output and the monopoly solution is $(a-c)/2b$ units of output

Nash (Cournot) Equilibrium

Profits of a firm

$$\pi_i = (a - c - b\sum_j Q_j)Q_i$$

Best response of a firm

$$\partial\pi_i/\partial Q_i = (a - c) - b\sum_j Q_j - bQ_i = 0$$

NOW and only NOW we use the equilibrium condition

symmetry: $Q_i = (1/n)Q$

plug in and solve

$$(a - c) - bQ - (b/n)Q = 0$$

$$Q = \frac{n}{n+1}\frac{a-c}{b}$$

Characteristics of the Equilibrium

$$Q = \frac{n}{n+1}\frac{a-c}{b}$$

when $n = 1$ this gives the usual monopoly solution

as $n \to \infty$ this approaches the competitive solution

Concepts

- coordination game
- **Nash equilibrium**
- **best response**, reaction function
- **oligopoly**
- **Cournot equilibrium**

Skill

Given a matrix game

find the Nash equilibrium

Given information about consumer utility and the costs of firms

find the Cournot equilibrium