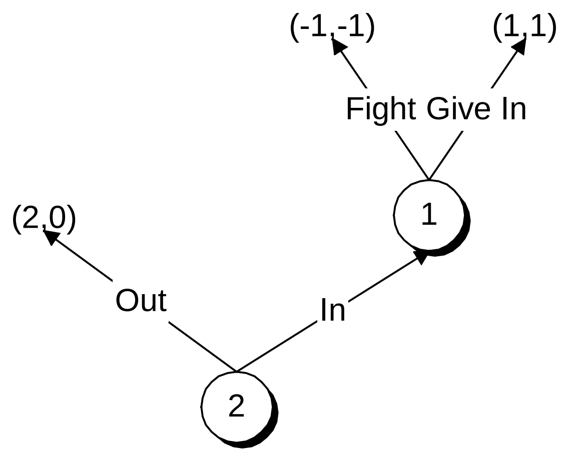
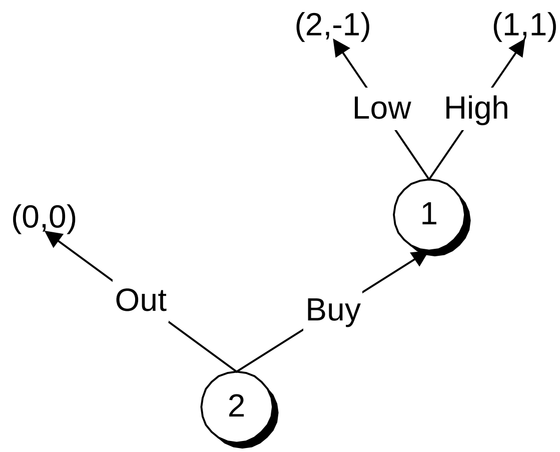
Reputation

Extensive Form Examples



Chain Store Game



Quality Game

Simultaneous Move Examples

Modified Chain Store

	out	in
fight	2-ε, 0	-1,-1
give in	2,0	1,1**

Inflation Game

	Low	High
Low	0,0	-2,-1
High	1,-1	-1,0

Inflation Game: LR=government, SR=consumers

consumer preferences are whether or not they guess right

	Low	High
Low	0,0	0,-1
High	-1,-1	-1,0

with a hard-nosed government

The Model

multiple types of long-run player $\omega \in \Omega$

Ω is a countable set of types

type is fixed forever (does not change from period to period)

$u^1(a,\omega)$ utility depends on type

strategy $\sigma^1(h,\omega)$ depends on type

types are privately known to long-run player, not known to short run player

strategy $\sigma^2(h)$ does not depend on type

μ probability distribution over Ω commonly known short-run player prior over types

Truly Committed Types

type $\omega(a^1)$ has a dominant strategy to play a^1 in the repeated game:

$$u^1(\tilde{a}^1, a^2, \omega(a^1)) = \begin{cases} 1 & \tilde{a}^1 = a^1 \\ 0 & \tilde{a}^1 \neq a^1 \end{cases}$$

for example

A truly committed type “can't be bargained with…can't be reasoned with…doesn't feel pity, or remorse, or fear.”

Let $n(\omega)$ be the least utility received by a type ω in any Nash equilibrium

let a^1* be a pure strategy Stackelberg strategy for type ω_0, with corresponding utility

$$u^1* = \max_{\alpha^1} \min_{\alpha^2 \in BR(\alpha^1)} u^1(\alpha^1, \alpha^2, \omega_0)$$

Theorem: Fix ω_0 with $\mu(\omega_0) > 0$, and. Let $\omega* \equiv \omega(a^1*)$, and suppose that $\mu* \equiv \mu(\omega*) > 0$. Then there is a constant $k(\mu*)$ otherwise independent of μ, Ω such that

$$n(\omega) \geq \delta^{k(\mu*)} u^1* + (1 - \delta^{k(\mu*)}) \underline{u}^1$$

Proof

define $\pi_t *$ to be the probability at the beginning of period *t* by the short-run player that he is facing type $\omega *$

Let $N(\pi_t * \leq \overline{\pi})$ be the number of times $\pi_t * \leq \overline{\pi}$

Lemma 1: Suppose that LR plays $a^1 *$ always. Then for any history *h* that has positive probability

$$pr(N(\pi_t * \leq \overline{\pi}) > \log \mu * / \log \overline{\pi} | h) = 0$$

Lemma 2: There is $\overline{\pi} < 1$ such that if $\pi_t * > \overline{\pi}$ the SR player plays a best response to $a^1 *$

- Why do these Lemma’s imply the theorem?

- Why is Lemma 2 true?

Proof of Lemma 1

Bayes Law

$$\pi(\omega^* | h_t) = \frac{\pi(\omega^* | h_{t-1})\pi(h_t | \omega^*, h_{t-1})}{\pi(h_t | h_{t-1})}$$

given h_{t-1} player 1 and 2 play independently

$$\pi(\omega^* | h_t) = \frac{\pi(\omega^* | h_{t-1})\pi(h_t | \omega^*, h_{t-1})}{\pi(h_t^1 | h_{t-1})\pi(h_t^2 | h_{t-1})}$$

since player 1's type isn't known to player 2

$$\pi(\omega^*|h_t) = \frac{\pi(\omega^*|h_{t-1})\pi(h_t|\omega^*, h_{t-1})}{\pi(h_t^1|h_{t-1})\pi(h_t^2|\omega^*, h_{t-1})}$$

since player 1's strategy is to always play a^1* $\pi(h_t^1|\omega^*, h_{t-1}) = 1$ so

$$\pi(\omega^*|h_t) = \frac{\pi(\omega^*|h_{t-1})\pi(h_t^2|\omega^*, h_{t-1})}{\pi(h_t^1|h_{t-1})\pi(h_t^2|\omega^*, h_{t-1})}$$

$$= \frac{\pi(\omega^*|h_{t-1})}{\pi(h_t^1|h_{t-1})} = \frac{\pi(\omega^*|h_{t-1})}{\pi_t^*}$$

the conclusion:

$$\pi(\omega^* | h_t) = \frac{\pi(\omega^* | h_{t-1})}{\pi_t^*}$$

- what does this say?

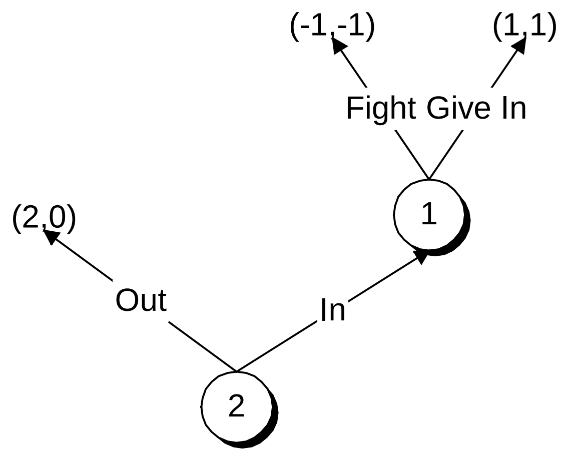
the Lemma now derives from the fact that $\pi(\omega^* | h_t) \leq 1$

Observational Equivalence

$\rho(y|\alpha)$ outcome function

$\alpha^2 \in W(\alpha^1)$ if there exists $\tilde{\alpha}^1$ such that $\rho(\cdot|\tilde{\alpha}^1, \alpha^2) = \rho(\cdot|\alpha^1, \alpha^2)$ and $\alpha^2 \in BR(\tilde{\alpha}^1)$

$$u^1 * = \max_{\alpha^1} \min_{\alpha^2 \in W(\alpha^1)} u^1(\alpha^1, \alpha^2, \omega_0)$$



Chain Store Game

strategies that are observationally equivalent

	out	in	mixed
fight	all	fight	fight
give	all	give	give
mixed	all	mixed	mixed

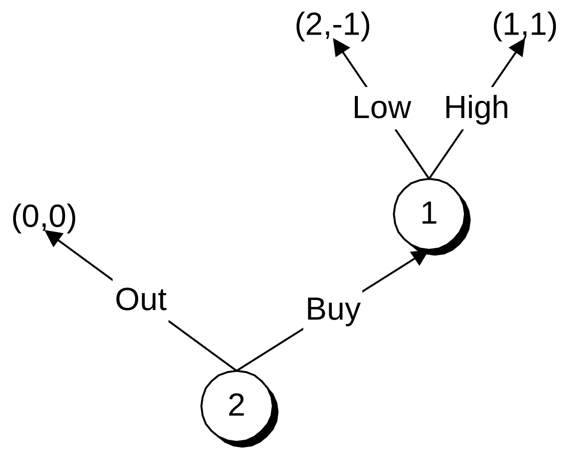
weak best responses

fight: out

give: in, out

mixed: in, out?

Best case fight:out so $u^1* = 2$



Quality Game

strategies that are observationally equivalent

	out	buy	mixed
hi	all	hi	hi
lo	all	lo	lo
mixed	all	mixed	mixed

weak best responses

hi: in, out

lo: out

mixed: in?, out

in every case out is a weak best response so $u^{1*} = 0$

Moral Hazard and Mixed Commitments

$\rho(y|\alpha)$ outcome function

expand space of types to include types committed to mixed strategies: leads to technical complications because it requires a continuum of types

$p(h_{t-1})$ probability distribution over outcomes conditional on the history (a vector)

$p^+(h_{t-1})$ probability distribution over outcomes conditional on the history and the type being in Ω^+

Theorem: for every $\varepsilon > 0, \Delta_0 > 0$ and set of types Ω^+ with $\mu(\Omega^+) > 0$ there is a *K* such that if Ω^+ is true there is probability less than ε that there are more than *K* periods with

$$\left\| p^+(h_{t-1}) - p(h_{t-1}) \right\| > \Delta_0$$

look for tight bounds

let $\underline{n}, \overline{n}$ be best and worst Nash payoffs to LR

try to get

$$\liminf_{\delta \to 1} \underline{n}(\omega) = \limsup_{\delta \to 1} \overline{n}(\omega) = \max_{\alpha^2 \in BR(\alpha^1)} u^1(\alpha)$$

game is *non-degenerate* if there is no undominated pure action a^2 such that for some $\alpha^2 \neq a^2$

$$u^i(\cdot, a^2) = u^i(\cdot, \alpha^2)$$

counterexample: player 2 gets zero always, player 1 gets either zero or one depending only on player 2's action game is *identified* if for all α^2 that are not weakly dominated $\rho(\cdot|\alpha^1,\alpha^2)=\rho(\cdot|\tilde{\alpha}^1,\alpha^2)$ implies $\alpha^1=\tilde{\alpha}^1$

$$\rho(\cdot|\alpha^1,\alpha^2)=\alpha^1 R(\alpha^2)$$

condition for identification $R(\alpha^2)$ has full row rank for all α^2

Patient Short Run Players: Schmidt

short run preferences $\begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}$

long run preferences

$\mu^0 = 0.1$ $\begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}$ pure coordination

$\mu^* = 0.01$ $\begin{bmatrix} 10 & 10 \\ 0 & 1 \end{bmatrix}$ commitment type

$\mu^i = 0.89$ $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ indifferent type strategies:

normal: play U except if you previously did D, then switch to D

commitment: always play U

indifferent type: U until deviation then D

SR: play L then alternate between R and L (on path)

if 1 deviated to D switch to R forever

if 2 deviated play L; if 1 reacts with U continue with L

reacts with D continue with R

$\delta_1 \geq .15, \delta_2 \geq .75$ then this is a subgame perfect equilibrium

- interesting deviation for SR when supposed to do R deviate to L; but then indifferent type switches to D forever
- for the normal type to prove he's not type "i" he must play D revealing he is not the commitment type

Suppose that LR can minmax SR in a pure strategy $\underline{a}^1$

Theorem: LR gets at least $\min_{\alpha^2 \in BR^2(\underline{a}^1)} u^1(\underline{a}^1, \alpha^2)$

let $\underline{u}^2$ be SR minmax

let $\tilde{u}^2$ be second best against $\underline{a}^1$

$$N = \frac{\ln(1-\delta_2) + \ln(\underline{u}^2 - \tilde{u}^2) - \ln(\overline{u}^2 - \tilde{u}^2)}{\ln \delta_2}$$

$$\varepsilon = \frac{(1-\delta_2)^2(\underline{u}^2 - \tilde{u}^2)}{(\overline{u}^2 - \tilde{u}^2)} - {\delta_2}^N(1-\delta_2)$$

commit to $\underline{a}^1$

Lemma: suppose $a_2^{t+1} \notin BR(\underline{a}^1)$ with positive probability, then SR must believe that in *t*+1,…,*t*+*N* there is a probability of at least ε of not having $\underline{a}^1$

- why is this sufficient?

Proof of Lemma:

2 can get at least $\underline{u}^2$ so

$$(1-\delta_2)u(\alpha^1, a_2^{t+1}) + \delta_2 V \geq \underline{u}^2$$

if $pr(\underline{a}^1) > 1-\varepsilon$ in *t*+1,…,*t*+*N*

suppose the optimum is not a br; consider deviating to a br

gain at least $\underline{u}^2 - \tilde{u}^2$ at *t*+1; starting at *t*+2 earn no less than $\underline{u}^2$

if you hadn't deviated, starting at *t*+2 would have earned at most

$$(1-\delta_2)\sum_{t=1}^{N} \delta_2^t(\underline{u}^2(1-\varepsilon) + \varepsilon\overline{u}^2) + \delta_2^{N+1}\overline{u}^2$$

but we chose *N* and ε so that the loss exceeds the gain