Economic 211B, David K. Levine
Answers to Problems on Reputation

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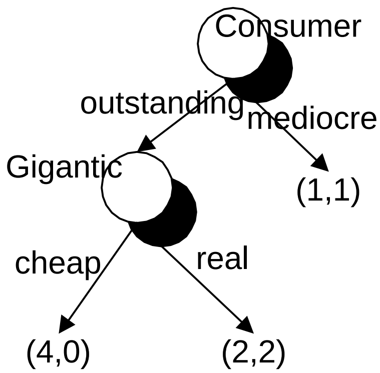
1. Reputation

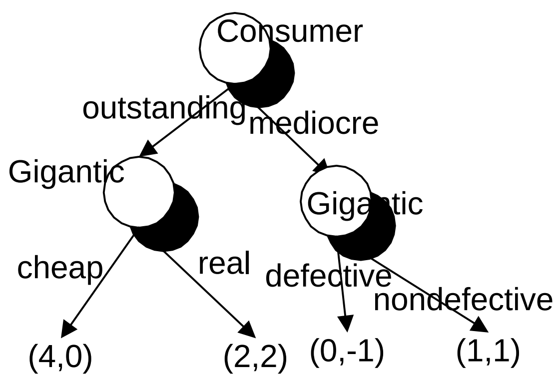
Gigantic = player 1



unique subgame perfect equilibrium is cheap: mediocre

(a) cheap: mediocre must also be an equilibrium in the repeated game

(b) Gigantic cannot prove it produces “real” if probability of “honest” corporation is so low the consumer won’t try the outstanding product



(c)Again, cheap: non-defective: mediocre is unique subgame perfect in game played once, so also an equilibrium in repeated game

(d) now producing “defective” products forces the consumer to believe that defective products will be produced in the future; Gigantic can then successfully imitate a type that produces defective cheap produce; the best response against such a type by the consumer is to buy the cheap product, giving gigantic a utility of 4.

2. Inference and Martingales

(a) both p_t, q_t are probability perceived before the fact of the event that occurred at time *t;* p_t is conditional on type ω^+; q_t is condition on type not ω^+.

(b) by inductive hypothesis

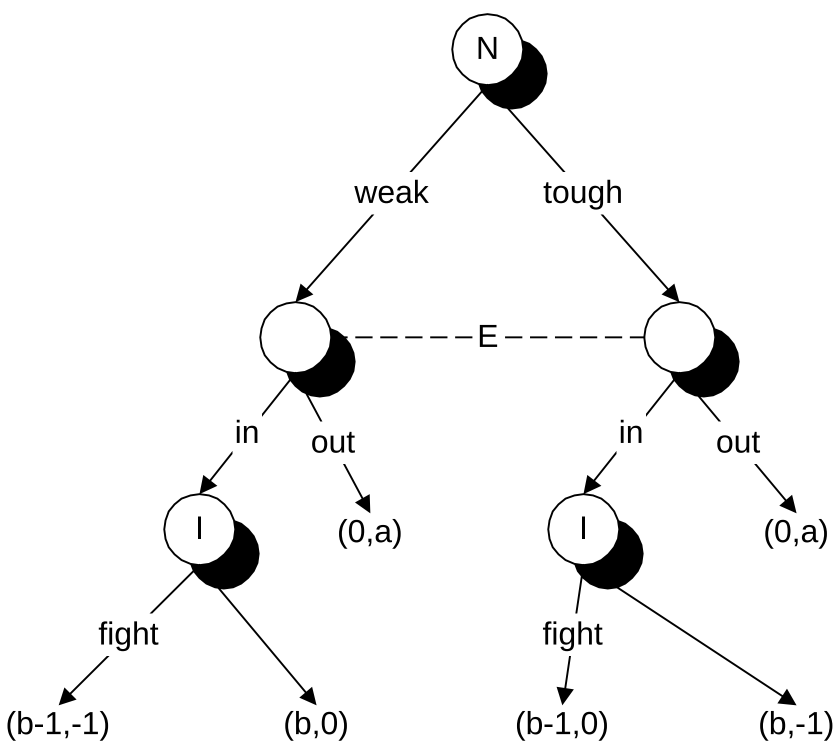
$$L_t = \frac{prob(a_t|h_{t-1},\Omega^+)\,prob(\Omega^+|h_{t-1})}{prob(a_t|h_{t-1},\omega^+)\,prob(\omega^+|h_{t-1})}$$
$$= \frac{prob(a_t,\Omega^+|h_{t-1})\,prob(a_t)}{prob(a_t,\omega^+|h_{t-1})\,prob(a_t)}$$
$$= \frac{prob(\Omega^+|a_t,h_{t-1})}{prob(\omega^+|a_t,h_{t-1})}$$

(c) Let $L_{t-1}, h_{t-1\ldots}$ be fixed and let $\sigma(\omega,a) \equiv \sigma_t(h_{t-1},\omega)(a); \mu^+(\omega) \equiv \mu(\omega|h_{t-1})$

$$EL_t = \left[E\,\frac{q_t}{p_t}\right]L_{t-1}$$
$$= \left[\frac{\sum_{\sigma(\omega^+,a)>0} \dfrac{\sum_{\omega\in\overline{\Omega}} \mu^+(\omega)\sigma(\omega,a)}{1-\mu^+(\omega^+)}}{\sigma(\omega^+,a)}\sigma(\omega^+,a)\right]L_{t-1}$$
$$= \left[\frac{\sum_{\sigma(\omega^+,a)>0}\sum_{\omega\in\Omega^+}\mu^+(\omega)\sigma(\omega,a)}{\sum_{\omega\in\Omega^+}\mu(\omega)}\right]L_{t-1}$$

but $\sum_{a\in A}\sum_{\omega\in\Omega^+}\mu(\omega)\sigma(\omega,a) = \sum_{\omega\in\Omega^+}\mu(\omega)$ so $\sum_{\sigma(\omega^+,a)}\sum_{\omega\in\Omega^+}\mu(\omega)\sigma(\omega,a) \le \sum_{\omega\in\Omega^+}\mu(\omega)$

3. Chain Store Paradox Paradox



Two period equilibrium
entrants beliefs: fight implies weak incumbent

After entry: strong incumbent get

	period 1	period 2
acquiesce	-1	a
fight	0	0 (since entrant believes you are weak)

So the strong incumbent will acquiesce; Note that unless a fight occurs in period 1 entry never occurs in period two since $\gamma > 1$