Economic 211, David K. Levine
Problems on Learning

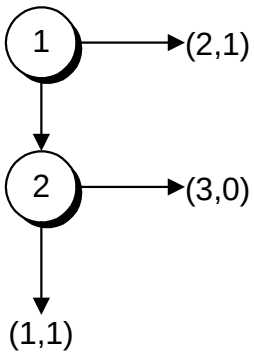
Last modified: March 16, 1999 **(click here for answers)**

1. In a two-player game, a marginal best response distribution is a probability distribution over outcomes (possibly correlated) such that each player has expected utility at least as much as that from the best response to the marginal distribution over his opponents actions. Show that in a 2x2 game a marginal best best response distribution is a correlated equilibrium. (WARNING: this is pretty hard.)

2. Starting with an initial condition in which each player has observed one heads, calculate the first ten periods of fictitious play in matching pennies. Assume that in case of a tie, a player plays the same way he did the previous period. What is the frequency each player played heads?

3. Consider a two state Markov process in which there is a 3/4 chance of remaining in state 1 but only a 1/2 chance of remaining in state 2. What is the unique stationary distribution? What does this mean about the long run frequency with which state 1 is observed?

4. Find all of the heterogeneous self-confirming equilibria of the extensive form game below:



5. Consider the one population continuous time replicator dynamic for the symmetric game with payoffs

$$\begin{bmatrix} 1,1 & 2,0 & 0,2 \\ 0,2 & 1,1 & 2,0 \\ 2,0 & 0,2 & 1,1 \end{bmatrix}$$

   (a) Find the unique Nash equilibrium of this game.

   (b) Find the eigenvalues for the replicator at that Nash equilibrium.

(c) What does this tell you about the dynamical system?

6. Consider the "exponential" best response dynamic for a heterogeneous population model

$$\dot{\theta}_i(s_i) = \frac{\exp(\kappa_i u_i(s_i, \theta_{-i}))}{\sum_{\tilde{s}_i} \exp(\kappa_i u_i(\tilde{s}_i, \theta_{-i}))} - \theta_i(s) \ .$$

(a) What happens as $\kappa_i \to \infty$?

(b) Show that for any $\kappa_i < \infty$ this dynamical system is volume contracting (hint: linearize the system; it suffices to show that the trace of the resulting matrix is negative).

(c) What does the fact that the system is volume contracting tell you about the dynamics of the system?