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Learning in Games

by David K. Levine Last modified: September 1, 2005



we are going to focus on longer term learning and implications for "steady states"

active vs. passive learning

for active learning players must be patient so willing to undertake investment

active learning leads at least to Nash, maybe even refinement (still an unresolved issue)

also study of particular cases such as risk dominant equilibrium

Self Confirming Equilibrium

 $s_i \in S_i$ pure strategies for *i*; $\sigma_i \in \Sigma_i$ mixed

 H_i information sets for *i*

 $\overline{H}(\sigma)$ reached with positive probability under σ

 $\pi_i \in \Pi_i$ behavior strategies

 $\hat{\pi}(h_i | \sigma_i)$ map from mixed to behavior strategies

 $\hat{\rho}(\pi)$, $\hat{\rho}(\sigma) \equiv \hat{\rho}(\hat{\pi}(\sigma))$ distribution over terminal nodes

 μ_i a probability measure on Π_{-i}

 $u_i(s_i | \mu_i)$ preferences

$$\Pi_{-i}(\sigma_{-i} | J) \equiv \{ \pi_{-i} | \pi_i(h_i) = \hat{\pi}(h_i | \sigma_i), \forall h_i \in H_{-i} \cap J \}$$

Notions of Equilibrium Nash equilibrium

a mixed profile σ such that for each $s_i \in \text{supp}(\sigma_i)$ there exist beliefs μ_i such that

- s_i maximizes $u_i(\cdot|\mu_i)$
- $\mu_i(\Pi_{-i}(\sigma_{-i}|H)) = 1$

Unitary Self-Confirming Equilibrium

• $\mu_i(\Pi_{-i}(\sigma_{-i}|\overline{H}(\sigma))) = 1$

(=Nash with two players)

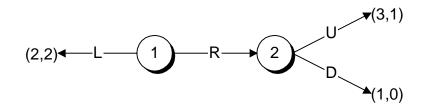
Heterogeneous Self-Confirming equilibrium

$$\mu_i(\Pi_{-i}(\sigma_{-i}|\overline{H}(s_i,\sigma))) = 1$$

Can summarize by means of "observation function"

 $J(s_i, \sigma) = H, \overline{H}(\sigma), \overline{H}(s_i, \sigma)$

Public Randomization



Remark: In games with perfect information, the set of heterogeneous self-confirming equilibrium payoffs (and the probability distributions over outcomes) are convex