# Midterm Exam: Economics 201b
# Answer Key

February 11, 2005

## Question 1

a) Normal form and best responses (in bold)

| | u | d |
|---|---|---|
| LL | 2.5,**2.5** | **2.5**,**2.5** |
| LR | 4,**2.5** | 2,**2.5** |
| RL | 4,**1.5** | 2,1 |
| RR | **5.5**,**1.5** | 1.5,1 |

$(LL, d)$ is a NE.

b) The only subgame is the game itself. Therefore, all NE of the game are subgame perfect.

c) No, $(LL, d)$ is not THP. Take any $\sigma_1^n \to (1,0,0,0)$ where $\sigma_1^n = (\sigma_{LL}^n, \sigma_{LR}^n, \sigma_{RL}^n, \sigma_{RR}^n) >> 0$ for all $n$. Then, $u_2(u, \sigma_1^n) = 2.5 \times (\sigma_{LL}^n + \sigma_{LR}^n) + 1.5 \times (\sigma_{RL}^n + \sigma_{RR}^n)$ and $u_2(d, \sigma_1^n) = 2.5 \times (\sigma_{LL}^n + \sigma_{LR}^n) + 1 \times (\sigma_{RL}^n + \sigma_{RR}^n)$. Thus, $u_2(u, \sigma_1^n) > u_2(d, \sigma_1^n)$ for any $n$ since $\sigma_{RL}^n + \sigma_{RR}^n > 0$. Hence, $d$ is not a best response to any tremble by player 1.

d) Denote the assessment by player 2 as $\alpha_2 = (\alpha, 1-\alpha)$. The probability of being at the upper node is $\alpha$ and the probability of being at the lower node is $1-\alpha$. Then, $u_2(u|\alpha_2) = 1 \times \alpha + 2 \times (1-\alpha) = 2 - \alpha$. Likewise, $u_2(d|\alpha_2) = 0 \times \alpha + 2 \times (1-\alpha) = 2 - 2\alpha$. Playing $d$ is optimal only if $\alpha = 0$.

e) Yes, the assessments are consistent. Let player 1 tremble in the following way: $\pi_1^n = \left(\pi_{L|U}^n, \pi_{R|U}^n, \pi_{L|D}^n, \pi_{R|D}^n\right) = (1-\varepsilon_u^n, \varepsilon_u^n, 1-\varepsilon_d^n, \varepsilon_d^n)$. By Bayes Rule, the assessment of the upper node is

$$\alpha^n = \frac{\varepsilon_u^n}{\varepsilon_u^n + \varepsilon_d^n} = \frac{1}{1 + \frac{\varepsilon_d^n}{\varepsilon_u^n}}$$

For $\alpha^n \to 0$, we need $\frac{\varepsilon_d^n}{\varepsilon_u^n} \to \infty$. Pick $\varepsilon_d^n = \frac{1}{2n}$ and $\varepsilon_n^n = \frac{1}{2n^2}$. Then,

$$\frac{\varepsilon_d^n}{\varepsilon_u^n} = \frac{\frac{1}{2n}}{\frac{1}{2n^2}} = n$$

and

$$\alpha = \lim_{n\to\infty} \alpha^n = \lim_{n\to\infty} \frac{1}{1+n} = 0$$

## Question 2

a) Game tree

b) SPNE is $(GG, G)$.

c) Normal form

| | G | P |
|---|---|---|
| GG | 1,0 | 1,0 |
| GP | 1,0 | 1,0 |
| PG | 0,2 | 4,0 |
| PP | 0,2 | 0,8 |

Best responses

| | G | P |
|---|---|---|
| GG | **1,0** | 1,**0** |
| GP | **1,0** | 1,**0** |
| PG | 0,**2** | **4**,0 |
| PP | 0,2 | 0,**8** |

Two pure NE: $(GG, G)$ and $(GP, G)$. Show that no mixed NE involving PG or PP.

Mixed strategies cannot use strictly dominated strategies and PP is strictly dominated by GG for player 1. Hence, PP is not part of a NE. Player 1 will put positive probability on PG only if player 2 puts at least some probability on P. Player 2 will only put positive probability on P if player 1 puts zero probability on PG. Therefore, PG cannot be part of a mixed equilibrium.

d) Yes. For example, any mixing of GG and GP against G.

e) Reduced normal form and best responses

| | G | P |
|---|---|---|
| G | **1**,**0** | 1,**0** |
| PG | 0,**2** | **4**,0 |
| PP | 0,2 | 0,8 |

Pure NE: $(G, G)$. Semi-mixed: $\sigma_1 = (1,0,0)$ and $\sigma_2 = (p, 1-p)$ for $p \geq \frac{3}{4}$.

Check: $u_1((1,0,0),(p,1-p)) = 1$, $u_1((0,1,0),(p,1-p)) = 4(1-p) = 4-4p$. $u_1((1,0,0),(p,1-p)) \geq u_1((0,1,0),(p,1-p))$ iff $1 \geq 4-4p$ iff $p \geq \frac{3}{4}$.

f) Normal type and his best responses

| | G | P |
|---|---|---|
| G | **1**,0 | 1,0 |
| PG | 0,2 | **4**,0 |
| PP | 0,2 | 0,8 |

Deviant type and his best responses

| | G | P |
|---|---|---|
| G | 1,0 | 1,0 |
| PG | **2**,2 | 4,0 |
| PP | **2**,2 | **8**,8 |

Candidate for BNE:

Normal player 1 plays PG, Deviant type plays PP, player 2 plays P.

Check: Both of player 1's types are playing a BR to P. Player 2 gets utility $u_2(P) = 0.7 \times 0 + 0.3 \times 8 = 2.4 > 2 = u_2(G)$. Thus, it is a BNE.

g) Is there BNE with $\Pr(G) > 0$? Deviant type never grabs in the first period because it is strictly dominated by PG. Normal type may grab if player 2 grabs sufficiently often.

Candidate: Normal player 1 plays G, Deviant type plays PG, player 2 plays G.

Check: Both of player 1's types are playing a BR to G. Given that both types of player 1 will eventually grab, the only way for player 2 of getting a positive payoff is always grabbing.