1. Short Answers

For each of the normal form games below, find all of the Nash equilibria. Which are Pareto Efficient?

a)

	L	R
U	10*,10* (Not efficient)	14*,0
D	0,14*	12,12

Nash Equilibria:

(U,L): Not efficient

b)

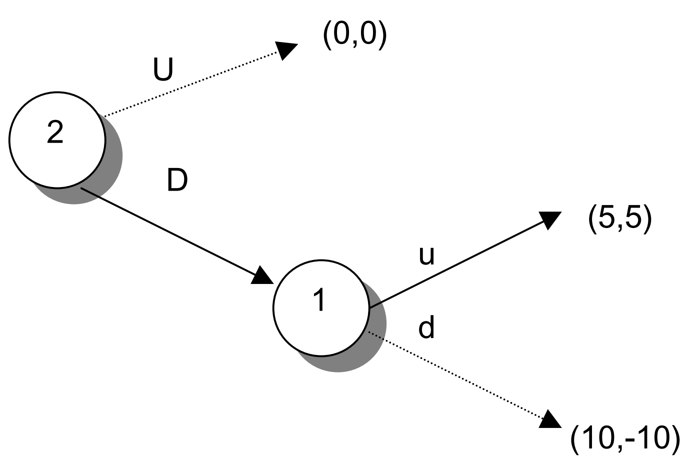
	L	R
U	4,0	5*,2* (Efficient)
D	6*,1* (Efficient)	4,0

Nash Equilibria:

(D,L): Efficient
(U,R): Efficient

For each of the extensive form games below, find the normal form and all Nash equilibria. Then find all of the subgame perfect equilibria. Which are Pareto Efficient?

c) Extensive form with subgame perfect choices marked with dashed lines



Subgame perfect equilibria:

(d, U): Not efficient

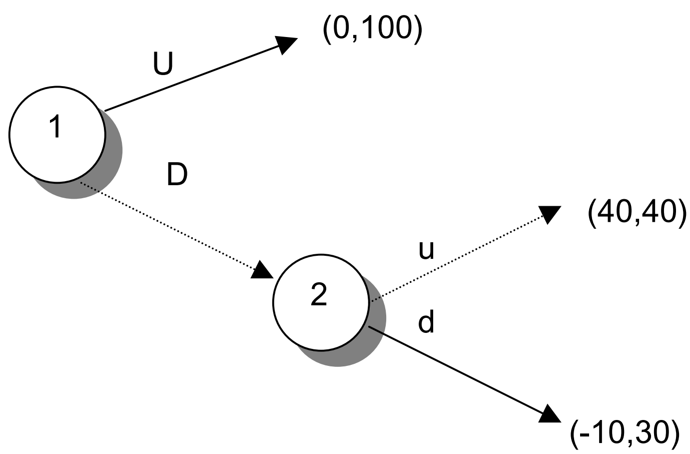
normal form with best response correspondence and Nash equilibria marked

	U	D
u	0*,0	5,5*
D	0*,0* (Not Efficient)	10*,-10

Nash equilibria:

(d, U): Not Efficient

d) Extensive form with subgame perfect choices marked with dashed lines



Subgame perfect equilibria:

(D, u): Efficient

normal form with best response correspondence and Nash equilibria marked

	u	d
U	0,100*	*0,100* (Efficient)
D	*40,40* (Efficient)	-10,30

Nash equilibria:

(D, u): Efficient
(U, d): Efficient

2. Cournot with Imperfect Substitutes

a) profit function for each firm:

- The profit function for firm 1 is:

$$\begin{aligned}\pi_1 &= p_1 x_1 - MC_1 x_1 \\ &= [17 - (x_1 + x_2)]x_1 - x_1 \\ &= 16x_1 - x_1(x_1 + x_2) \\ &= 16x_1 - x_1^2 - x_1 x_2\end{aligned}$$

- The profit function for firm 2 is:

$$\begin{aligned}\pi_2 &= p_2 x_2 - MC_2 x_2 \\ &= [(17 - (x_1 + x_2)) + b(17 - x_2)]x_2 - x_2 \\ &= x_2[16 - x_1 + b(17 - x_2) - x_2] \\ &= -(1 + b)x_2^2 - x_1 x_2 + (16 + 17b)x_2\end{aligned}$$

b) Nash equilibrium levels of output for each firm:

- Partial differentiation of π_1 with respect to x_1 is:

$$\frac{\partial \pi_1}{\partial x_1} = 16 - 2x_1 - x_2$$

  Set the above equation equal to zero. The best-response function for firm 1 is:

$$x_1 = 8 - \frac{x_2}{2}$$

- Partial differentiation of π_2 with respect to x_2 is:

$$\frac{\partial \pi_2}{\partial x_2} = 16 + 17b - x_1 - 2(1 + b)x_2$$

  Set the above equation equal to zero. The best-response function for firm 2 is:

$$x_2 = \frac{16 + 17b}{2(1 + b)} - \frac{x_1}{2(1 + b)}$$

- Nash equilibrium levels of output are the intersection of these two best-response functions:

$$x_1 = 8 - \frac{x_2}{2}$$
$$x_2 = \frac{16+17b}{2(1+b)} - \frac{x_1}{2(1+b)}$$

Hence, the Nash equilibrium levels of output for firm 1 and firm 2 are:

$$x_1^* = \frac{16+15b}{3+4b}$$
$$x_2^* = \frac{2(8+17b)}{3+4b}$$

c) When the quality of good 2 improves (i.e. when b gets larger):

- Partial differentiation of x_1^* with respect to b is:

$$\frac{\partial x_1^*}{\partial b} = \frac{-19}{(3+4b)^2} \quad < \quad 0$$

Hence, when b increases, x_1^* decreases. That is, when the quality of good 2 improves, the Nash equilibrium level of output for firm 1 decreases.

- Partial differentiation of x_2^* with respect to b is:

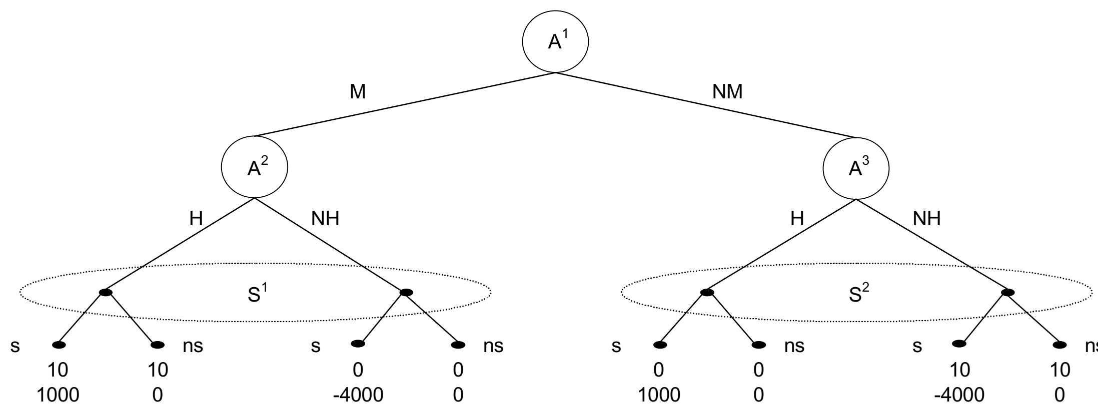
$$\frac{\partial x_2^*}{\partial b} = \frac{38}{(3+4b)^2} \quad > \quad 0$$

Hence, when b increases, x_2^* increases. That is, when the quality of good 2 improves, the Nash equilibrium level of output for firm 2 increases.

3. How to operate airport security ? (Answer Key, Matias Iaryczower, May 7, 2002)

The actions available to player 1 (Ms.A) are whether to put on her make up (M) or not (NM) and whether to put the nailclipper in her handbag (H) or not (NH). The actions available to player 2 (Ms.S) are to search (s) or not (ns).

(a). Extensive Form



(b). Normal Form, and Nash Equilibria

Note that each strategy is a complete plan of action specifying a choice after every contingency. Therefore, the possible strategies of Ms.A are {M-H-H, M-H-NH, M-NH-H, M-NH-NH, NM-H-H, NM-H-NH, NM-NH-H, NM-NH-NH}, while the possible strategies for Ms. S are {s-s,s-ns,ns-s,ns-ns}. We say that two strategies are "equivalent" for a player if they have the same consequences over outcomes regardless of how their opponents play. The reduced normal form of the extensive form game is obtained leaving just one strategy of each equivalent groups. Therefore, we use (i) M-H to represent the equivalent strategies M-H-H and M-H-NH, (ii) M-NH to represent M-NH-H and M-NH-NH, (iii) NM-H to represent NM-H-H and NM-NH-H, and (iv) NM-NH to represent NM-H-NH and NM-NH-NH. The reduced normal form representation of the game is then:

		S (P2)			
		s-s	s-ns	ns-s	ns-ns
A (P1)	M-H	(10,1000) [P]	(10,1000) [P]	(10,0)	(10,0)
	M-NH	(0,-4000)	(0,-4000)	(0,0)	(0,0)
	NM-H	(0,1000)	(0,0)	(0,1000)	(0,0)
	NM-NH	(10,-4000)	(10,0) [NP]	(10,-4000)	(10,0) [NP]

A strategy profile s* is a Nash equilibrium if, for each player j, the choice of strategy s_j* is optimal when the other player is playing her component of s*, s_{-j}*; that is, no player has an incentive to unilaterally deviate from the equilibium strategy. The Nash equilibria of this game are therefore the strategy profiles {(M-H,s-s),(M-H,s-ns),(NM-NH,s-ns), (NM-NH,ns-ns)}. [1] For example, if Ms.A puts on her make up and puts the nailclipper in her handbag, always searching is a best response by MsS, while if Ms.S will search independently of whether Ms.A is wearing make up or not, putting on her make up and putting the nailclipper in the bag is a best response for Ms.A.

(c) An outcome is Pareto Efficient if there is no other feasible outcome that can improve the utility of one player without "hurting" the other. Of the four N.E. of this game, you can check that only (M-H,s-s) and (MH-s-ns) are Pareto Efficient.

(d) A strategy s_j weakly dominates a strategy s_j' for player j if for every strategy of player –j, s_j is always at least as good as s_j' for player j, and is sometimes strictly better (that is, there is a strategy of player –j under which the payoff that player j could get by playing s_j is strictly higher than that from playing s_j'). Using this definition, in a first round of elimination of (weakly) dominated strategies, we can eliminate M-NH and NM-H for Ms.A. In a second round, we can eliminate all but s-ns for Ms.S. We can proceed no further than this, so the remaining normal form is:

		S (P2)
		s-ns
A (P1)	M-H	(10,1000) [P]
	NM-NH	(10,0) [NP]

[1] That is, the following strategy profiles constitute a Nash Equilibrium: (i) Ms.A puts on her make up (M) and puts the nailclippers in her handbag (H), while Ms.S searches (s) both after observing that Ms.A has her make up or not. This results in a payoff of 10 to Ms.A and of 1000 to Ms.S., (ii) Ms.A puts on her make up (M) and puts the nailclippers in her handbag (H), while Ms.S searches (s) after observing that Ms.A has her make up, but not (ns) after observing she is not. This results in a payoff of 10 to Ms.A and of 1000 to Ms.S, (iii) Ms.A doesn't put on her make up (NM) and doesn't put the nailclippers in her handbag (NH), while Ms.S searches (s) after observing that Ms.A has her make up, but not (ns) after observing she is not. This results in a payoff of 10 to Ms.A and of 0 to Ms.S, (iv) Ms.A doesn't put on her make up (NM) and doesn't put the nailclippers in her handbag (NH), while Ms.S doesn't search (ns) both after observing that Ms.A has her make up or not. This results in a payoff of 10 to Ms.A and of 0 to Ms.S.