Practice Problems: More Game Theory Under Uncertainty

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1. More Drug Testing

A sports figure tests positive for a banned substance. Suppose that 40% of athletes in this sport use the substance, and that the test is wrong 10% of the time. Consider the following table of costs and benefits:

	Banned	Not banned
User	10	-10
Not User	-50	0

Should the sports figure be banned? What if the utility from banning a non-user is –150?

2. The Beer Quiche Game

A newcomer walks into a bar. With probability .4 he is tough, and with probability .6 he is a wimp. Once he is in the bar, he may order beer or quiche. If he is tough he gets utility 4 from beer and 2 from quiche; if he is a wimp he gets utility 2 from beer and 4 from quiche. In the bar is a redneck. The redneck cannot tell whether or not the newcomer is a wimp, but he can see whether the newcomer orders beer or quiche. After this the redneck must decide whether or not to start a fight. If he fights a wimp he gets a utility of 2. If he fights a tough guy he gets a utility of –4. If he doesn't fight he gets a utility of 0. The newcomer prefers not to fight: if there is a fight he gets a penalty of 3. Draw the extensive form of the game. Find the normal form. What are the pure strategy Nash equilibria?

3. Cournot with Uncertain Cost

Consider a Cournot Duopoly with demand $p = 17 - x$. There are two possible levels of cost: with probability p^1 cost is low and equal to 1. With probability $1 - p^1$ cost is high and is equal to 3. Assuming that each firm knows its own cost and these probabilities, in the Bayesian Nash equilibrium of the Cournot game, what are the equilibrium strategies of the two firms?

4. Price Discrimination

A firm wishes to sell either 1 or 2 units of a good to a consumer with uncertain demand. (The firm is not allowed to sell 0 units; there is no production cost.) With probability 1/2 the utility function of the consumer is

$$(1-p)x$$

where p is the price paid per unit and x is the number of units purchased. With probability 1/2 the utility is instead

$$(3-p)x\,.$$

Only the consumer knows his utility function. Formulate this as a mechanism design problem for the seller, and determine how he can best maximize his expected revenue.