

# Final Exam: Economics 101

You have three hours. Do all 5 questions; each has equal weight. Problems marked with an asterisk are harder. Good luck.

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## 1. Normal Form Games

In each of the following games

- i) find all of the pure strategy Nash equilibria
- ii) determine whether or not there is a mixed strategy Nash equilibrium, and if so, what it is
- iii) Which of these equilibria are Pareto efficient?
- iv) Do any of the pure strategy equilibria involve playing weakly or strictly dominated strategies?
- v) Sketch the socially feasible set.
- vi\*) Find a pure strategy that maximizes the payoff each player is guaranteed no matter how the other player plays (this is called the maxmin payoff, although it was sometimes incorrectly referred to as the minmax payoff). Also find a pure strategy that minimizes the payoff the other player can get (this is called the minmax payoff).
- vii\*) Find the mixed strategy maxmin (which is the same as the minmax) payoff for each player. Sketch the corresponding socially feasible individually rational set.

a)

	L	R
U	2,5	0,0
D	0,0	5,2

b)

	L	R
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U	-1,1	-3,3
D	-3,3	-1,1

c)

	L	R
U	7,7	0,8
D	8,0	1,1

## **2. Long Run versus Short Run**

	L	R
U	3,1	0,0
D	8,0	1,2

Suppose that this stage game is repeated between infinitely lived player 1 with discount factor equal to  $\delta$  and a sequence of short-lived player 2's. What are the pure strategy Nash equilibria of the stage game? What is the Stackelberg equilibrium of the stage game in which player 1 moves first? Propose a strategy and a discount factor  $\delta$  such that in equilibrium players end up playing UL.

## **3. Screening**

A recent college school graduate must decide whether to go get an MBA or to continue working as a car salesperson for a utility of 5. With probability .5 the graduate is a nerd, and with probability .5 he is normal. He knows whether or not he is a nerd, but business school recruiters do not. If he is a nerd it costs 1 unit of utility to get an MBA; if he is normal the cost is 3 units of utility. The business school recruiter must decide whether to offer a straight salary of 7 or a salary of 7 plus a commission worth 1 unit of utility to the nerd and 2 units of utility to the normal type (the cost to the recruiter is the same as the

value to the graduate). The recruiter gets revenue of 8 from a nerd and a revenue of 11 from the normal person. However, if a commission is paid, the recruiter gets a revenue of 15 from the normal type. The recruiter gets nothing from a graduate who doesn't get an MBA.

- a) Draw the extensive form of the game.
- b) Find the normal form.
- c) What are the pure strategy Nash equilibria?
- d) What is the mixed strategy Nash equilibrium? (Hint: it is useful to eliminate strictly dominated strategies first.)
- e\*) What is the behavior strategy corresponding to the mixed strategy equilibrium?
- f\*) In this mixed strategy equilibrium, what are the beliefs of the recruiter about the type of MBA graduate he gets? (You will need to use Bayes law to answer this.)

#### **4. Decision Analysis**

A sports commission must decide how much to pay in order to administer a steroid test to its athletes. Suppose that 10% of athletes use steroids. Suppose also that when the test is given to a steroid user there is a 5% chance of making an error, and if it is given to a non-steroid user there is a 10% chance of making an error. Suppose that the costs and benefits to the commission are given below

	Athlete banned	Athlete allowed to participate
Steroid user	0	-50
Non-user	-100	10

Should a person who tests positive be banned from competition? How much is the most the commission would be willing to pay for the test?

### **5. Cournot with Uncertain Cost**

Consider a Cournot Duopoly with demand  $p = 17 - x$ . There are two possible levels of marginal cost: with probability  $1/3$  marginal cost is low and equal to 1. With probability  $2/3$  marginal cost is high and is equal to 3. Assuming that each firm knows its own marginal cost and these probabilities, in the Bayesian Nash equilibrium of the Cournot game, what are the equilibrium strategies of the two firms?