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# The Repeated Prisoner's Dilemma

recall the prisoner's dilemma game

		Player 2	
		don't confess	confess
Player 1	don't confess	32,32	28,35
	confess	35,28	30,30

- This is a simultaneous move game with a unique Nash equilibrium, and a unique strictly dominant strategy solution at 30, 30.
- The unique non-cooperative solution is Pareto dominated by 32, 32
- with repeated play, incentives are changed by the possibility of punishments and rewards in the future.

## ***Intertemporal Preference***

$u_i(t)$  is the utility or payoff received by player  $i$  in period  $t$

the game is repeated indefinitely and that intertemporal preferences are given by average present value

$$U_i = (1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} u_i(t)$$

where the common discount factor  $\delta$  is between 0 and 1.

- a basic feature of repeated games: regardless of the discount factors, the repeated static equilibrium is a subgame perfect equilibrium of the repeated game

## ***Grim Strategies***

the *grim strategy* in the repeated game is

- cooperate in the first period
- cooperate in subsequent periods as long as all players have cooperated in every previous period
- cheat in any period in which some player has cheated in any previous period

suppose the other player plays the grim strategy

- payoff to cheating

$$(1 - \delta)(35 + 30\delta + 30\delta^2 \dots)$$
$$= (1 - \delta)35 + 30\delta = 35 - 5\delta$$

- payoff to cooperating

$$32$$

- optimal to cooperate if

$$32 \geq 35 - 5\delta \text{ or}$$

$$\delta \geq 3/5$$

- if  $\delta \geq 3/5$  both players playing the grim strategy is a subgame perfect equilibrium

*why subgame perfect?*