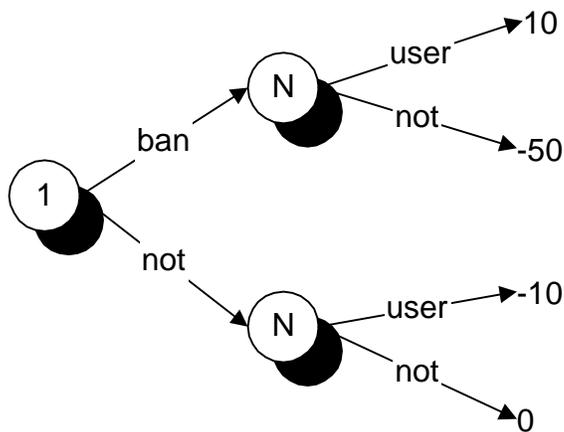


Practice Problems: More Game Theory Under Uncertainty

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1. Decision Analysis

Following a positive test, the decision tree is



We must use Bayes law to compute the probability of being a user or not given that the test is positive. Let U be the event that the athlete is a user, and P be the result that the test is positive

$$\Pr(U|P) = \frac{\Pr(P|U) \Pr(U)}{\Pr(P|U) \Pr(U) + \Pr(P|\sim U) \Pr(\sim U)} = \frac{.9 \times .4}{.9 \times .4 + .1 \times .6} = .86$$

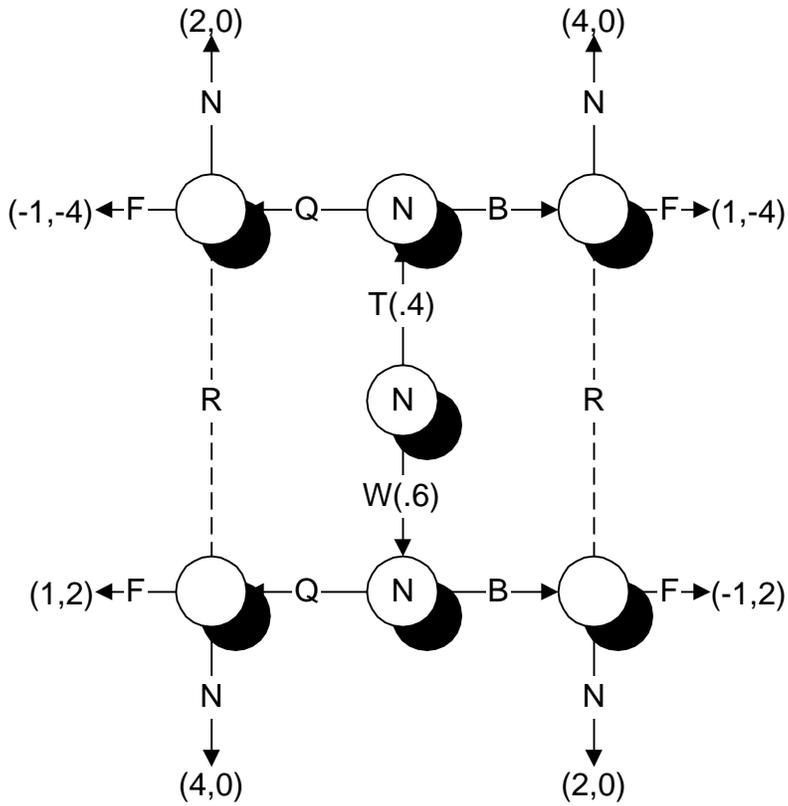
So the expected utility from banning is $.86 \times 10 + .14 \times (-50) = 1.6$

The expected utility from not banning is $.86 \times (-10) = -8.6$

So the correct decision is to ban. If the cost of banning an innocent athlete is -150 then the expected utility from banning is -12.4 so the athlete should not be banned.

2. Extensive Form Signaling

The extensive form is



Let strategies for the redneck (player 2) be denoted by FN where the first letter is the response to beer, the second to quiche; let strategies for the newcomer be BQ where the first letter is the response to tough and the second to wimp.

	FF	FN	NF	NN
BB	$-.2, -.4$	$-.2, -.4$	$2.8^*, 0^*$	$2.8, 0^*$
BQ	$1^*, -.4$	$2.8, -1.6$	$2.2, 1.2^*$	$4^*, 0$
QB	$-1, -.4$	$.2, 1.2^*$	$.8, -1.6$	$2, 0$
QQ	$.2, -.4$	$3.2^*, 0^*$	$.2, -.4$	$3.2, 0^*$

There are two pure strategy Nash equilibria: BB, NF and QQ, FN.

3. Cournot with Uncertain Cost

$$\begin{aligned}\pi_i(x_i, c_i) &= p^1 [17 - c_i - (x_i + x^1)] x_i \\ &\quad + (1 - p^1) [17 - c_i - (x_i + x^3)] x_i \\ &= [17 - c_i - (x_i + p^1 x^1 + (1 - p^1) x^3)] x_i\end{aligned}$$

maximize

$$\begin{aligned}\frac{d\pi_i(x_i, c_i)}{dx_i} &= [17 - c_i - (x_i + p^1 x^1 + (1 - p^1) x^3)] - x_i \\ &= [17 - c_i - (2x_i + p^1 x^1 + (1 - p^1) x^3)] = 0\end{aligned}$$

$$\text{so } x_i = (17 - c_i - p^1 x^1 - (1 - p^1) x^3) / 2$$

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solve each equation individually

$$x^1 (1 + p^1 / 2) = (16 - (1 - p^1) x^3) / 2$$

$$x^3 (3/2 - p^1 / 2) = (14 - p^1 x^1) / 2$$

plug the second into the first

$$x^1 (1 + p^1 / 2) = (16 - (1 - p^1) \frac{(14 - p^1 x^1) / 2}{(3/2 - p^1 / 2)}) / 2$$

solve

$$x^1 (2 + p^1) (3 - p^1) = 16(3 - p^1) - ((1 - p^1) 14 - (1 - p^1) p^1 x^1)$$

$$x^1 = \frac{16(3 - p^1) - ((1 - p^1) 14)}{(2 + p^1)(3 - p^1) - (1 - p^1) p^1} = \frac{17 - p^1}{3}$$

$$\text{where we have used } (1 - p^1 + 2)(p^1 + 2) - (1 - p^1) p^1 = 4 + 2(1 - p^1) + 2p^1 = 6$$

substitute back to get x^3

$$x^3 = \frac{(42 - p^1(17 - p^1))}{3(3 - p^1)} = \frac{(3 - p^1)(14 - p^1)}{3(3 - p^1)} = \frac{14 - p^1}{3}$$

4. Price Discrimination

$$(v^i - p^i) x^i \text{ utility of buyer}$$

$(v^i - p^i)x^i \geq 0$ individual rationality constraint

$(v^i - p^i)x^i \geq (v^i - p^{-i})x^{-i}$ incentive constraint

$v^i(x^i - x^{-i}) \geq p^i x^i - p^{-i} x^{-i}$ rewrite incentive constraint

$U = \pi^h x^h p^h + \pi^l x^l p^l$ utility of seller

guess that individual rationality binds on low value, incentive constraint on high value

$p^l = v^l$ IR low

$v^h(x^h - x^l) = p^h x^h - p^l x^l$ IC high

$v^h(x^h - x^l) = p^h x^h - v^l x^l$ combine them

$U = \pi^h (v^h(x^h - x^l) + v^l x^l) + \pi^l x^l v^l$ substitute in

case 1: $x^l = x^h$

then $U = x^l v^l = 2v^l$ and $p^h = p^l = v^l$

case 2: $x^l = 1, x^h = 2$

$U = \pi^h v^h + v^l$ $p^h = (v^h + v^l) / 2$

if $\pi^h v^h > v^l$ then price discriminate

otherwise don't

in particular if $v^h = 3, v^l = 1, \pi^h = 1/2$ then offer the following options

1 unit for a price of 1

or 2 units for a higher price of 2 (each)

Notice that this yields a profit of 2.5. If we are allowed to sell nothing to the low value type, then it would be better to charge a price of 3 for 2 units sold to the high value type for a profit of 3.

