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Discounting

Interest Rates and Discount Factors

interest at an annual rate of r

paid annually:

\$1 in the bank, and in one year collect $\$1+r$

discount factor:

to have \$1 in the bank in one year time, must put

$\delta = \frac{1}{1+r}$ in the bank today

a useful approximation

$$\frac{1}{1+r} \approx 1-r \text{ if } r \ll 1$$

r	$\frac{1}{1+r}$	$1-r$
1%	.9901	.9900
10%	.9091	.9000
50%	.6667	.5000

Present Value

1 dollar at the beginning of every year for τ years is worth what right now?

what is $z = 1 + \delta + \delta^2 + \dots + \delta^{\tau-1}$?

$$\delta z = z - 1 + \delta^\tau$$

$$(1 - \delta)z = 1 - \delta^\tau$$

$$1 + \delta + \delta^2 + \dots + \delta^{\tau-1} = z = \frac{1 - \delta^\tau}{1 - \delta}$$

Mortgage Interest

You buy a house for \$250,000. You make a 20% down payment, and get a 30 year fixed rate mortgage at 8% annual interest. How much are your monthly payments.

- suppose that monthly interest is $8\%/12=0.67\%$

- so $\delta = \frac{1}{1 + .0067} \approx .9933$

- mortgage is for \$200,000

- number of payments $\tau = 360$

let p be the monthly payment

then

$$200000 = (\delta + \delta^2 + \dots + \delta^{\tau})p = \delta \frac{1 - \delta^{\tau}}{1 - \delta} p$$

or

$$p = 200000 \frac{1}{\delta} \frac{1 - \delta}{1 - \delta^{\tau}}$$
$$\approx 200000 \frac{1}{.9933} \frac{.006655}{.9111} \approx 1471$$

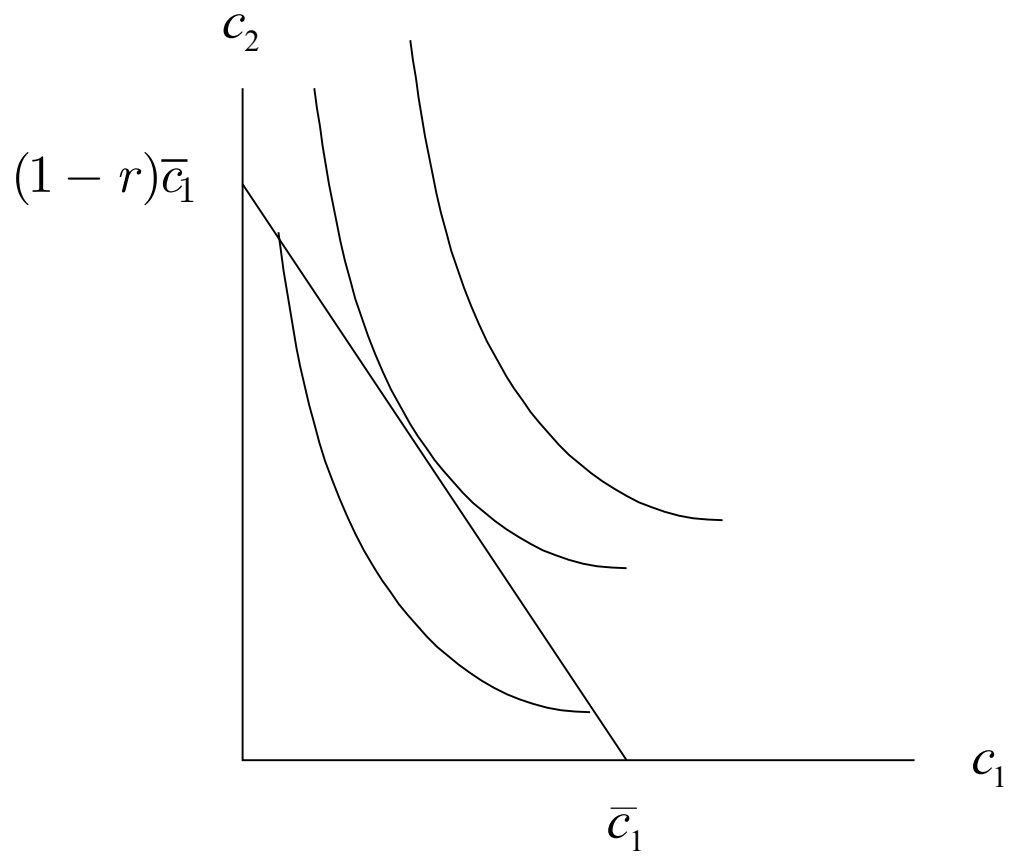
Interest and Intertemporal Prices

c_1 consumption in period 1

c_2 consumption in period 2

\bar{c}_1 endowment (period 1 only)

r interest rate from period 1 to period 2



The Napster Problem

demand $p = x^{-r}$ (note that price of 1 unit is 1)

revenue x^{1-r}

supply $x_t = \beta^{t-1}$

present value of revenue

$$\sum_{t=1}^{\infty} \delta^{t-1} (\beta^{t-1})^{1-r}$$

$$\sum_{t=1}^{\infty} (\delta\beta^{1-r})^{t-1} = 1 + (\delta\beta^{1-r}) + (\delta\beta^{1-r})^2 =$$

$$\frac{1}{1 - (\delta\beta^{1-r})}$$

if $r < 1$ then pv revenue $\rightarrow \infty$; if $r > 1$ then pv revenue $\rightarrow 1$

Intertemporal Preference in Repeated Games

u_t utility from the *stage game* in period t

- total utility

$$u_1 + u_2 + \dots + u_T$$

- finite horizon time average

$$\frac{u_1 + u_2 + \dots + u_T}{T}$$

- infinite horizon discounted utility

$$\sum_{t=1}^{\infty} \delta^{t-1} u_t = u_1 + \delta u_2 + \delta^2 u_3 + \dots$$

what happens as $\delta \rightarrow 1$?

- infinite horizon average discounted utility

$$\sum_{t=1}^{\infty} \delta^{t-1} = \frac{1}{1-\delta}, \text{ so use } (1-\delta) \sum_{t=1}^{\infty} \delta^{t-1} u_t$$