Discounting

Interest Rates and Discount Factors

interest at an annual rate of *r*

paid annually:

\$1 in the bank, and in one year collect \$1+r

discount factor:

to have \$1 in the bank in one year time, must put

$\delta = \frac{1}{1+r}$ in the bank today

a useful approximation

$$\frac{1}{1+r} \approx 1 - r \text{ if } r << 1$$

r	$\frac{1}{1+r}$	$1-r$
1%	.9901	.9900
10%	.9091	.9000
50%	.6667	.5000

Present Value

1 dollar at the beginning of every year for τ years is worth what right now?

what is $z = 1 + \delta + \delta^2 + \ldots + \delta^{\tau-1}$?

$\delta z = z - 1 + \delta^\tau$

$(1-\delta)z = 1 - \delta^\tau$

$$\boxed{1+\delta+\delta^2+\ldots+\delta^{\tau-1} = z = \frac{1-\delta^\tau}{1-\delta}}$$

Mortgage Interest

You buy a house for \$250,000. You make a 20% down payment, and get a 30 year fixed rate mortgage at 8% annual interest. How much are your monthly payments.

- suppose that monthly interest is 8%/12=0.67%
- so $\delta = \dfrac{1}{1 + .0067} \approx .9933$
- mortgage is for \$200,000
- number of payments $\tau = 360$

let *p* be the monthly payment

then

$$200000 = (\delta + \delta^2 + \ldots + \delta^\tau)p = \delta \frac{1-\delta^t}{1-\delta} p$$

or

$$\begin{aligned} p &= 200000 \frac{1}{\delta} \frac{1-\delta}{1-\delta^\tau} \\ &\approx 200000 \frac{1}{.9933} \frac{.006655}{.9111} \approx 1471 \end{aligned}$$

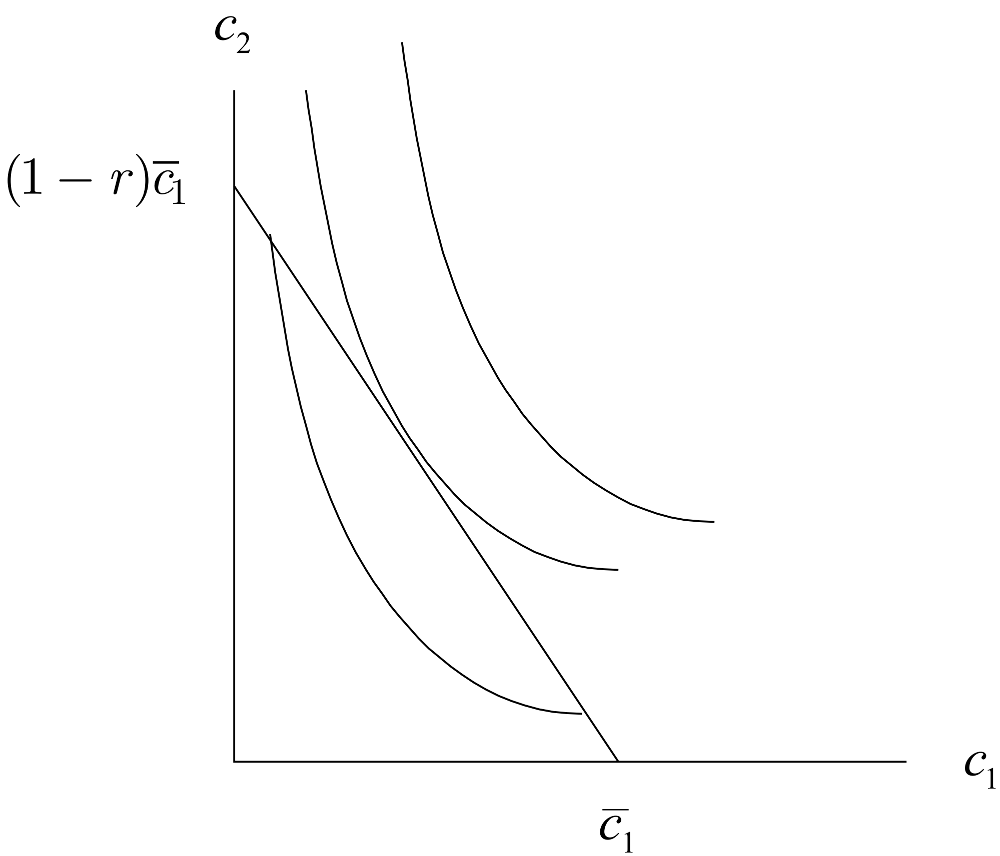
Interest and Intertemporal Prices

c_1 consumption in period 1

c_2 consumption in period 2

$\bar{c}_1$ endowment (period 1 only)

r interest rate from period 1 to period 2


c_2
$(1-r)\overline{c}_1$
c_1
$\overline{c}_1$

The Napster Problem

demand $p = x^{-r}$ (note that price of 1 unit is 1)

revenue x^{1-r}

supply $x_t = \beta^{t-1}$

present value of revenue

$$\sum_{t=1}^{\infty} \delta^{t-1}(\beta^{t-1})^{1-r}$$

$$\sum_{t=1}^{\infty} (\delta\beta^{1-r})^{t-1} = 1 + (\delta\beta^{1-r}) + (\delta\beta^{1-r})^2 =$$

$$\frac{1}{1-(\delta\beta^{1-r})}$$

if $r < 1$ then pv revenue $\rightarrow \infty$; if $r > 1$ then pv revenue $\rightarrow 1$

Intertemporal Preference in Repeated Games

u_t utility from the *stage game* in period *t*

- total utility

$u_1 + u_2 + \ldots + u_T$

- finite horizon time average

$$\frac{u_1 + u_2 + \ldots + u_T}{T}$$

- infinite horizon discounted utility

$$\sum_{t=1}^{\infty} \delta^{t-1} u_t = u_1 + \delta u_2 + \delta^2 u_3 + \ldots$$

what happens as $\delta \to 1$?

- infinite horizon average discounted utility

$\sum_{t=1}^{\infty} \delta^{t-1} = \frac{1}{1-\delta}$, so use $(1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} u_t$