

Twin Peaks: Expressive Externality in Group Participation*

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Abstract

We introduce a model of group behavior that combines expressive participation with strategic participation. Building on the idea that expressive voting in elections is much like rooting for a sports team (Brennan and Buchanan, 1984; Brennan and Hamlin, 1998), we give applications to both sporting events and elections. In our model there is an expressive externality: an individual enjoys an event more when more of her peers come out to support her preferred party or team. We show that this results in the possibility of “tipping” - that participation may jump up discontinuously when the externality becomes strong enough. We examine the implications for pricing by sports teams and for voter turnout.

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1. Introduction

In theory large groups involved in the production of public goods face a severe free riding problem, but in practice the problem often seems at least partially overcome - think for example of the voters who turn out to vote though they are not pivotal in the election.⁴ One force counteracting the incentive to free ride is an ethical motivation,⁵ with its more recent incarnation as the peer pressure model (Ali and Lin, 2013; Levine and Mattozzi, 2020).⁶ The other is an expressive motive - the desire to express one's views or feelings (Brennan and Buchanan, 1984; Brennan and Hamlin, 1998; Hillman, 2010).⁷ The present paper explores the case where both forces operate, and adds the new element of expressive externality. Our starting point is that it seems reasonable to suppose that a group member's utility depends on how many peers are expressing themselves - that, in other words, there is a positive expressive externality. Such an externality is present in voting and in other seemingly unrelated but substantially analogous contexts, such as rooting for a sports team: whether she participates or not, an individual typically enjoys an event more when she sees more of her peers coming out to support her preferred party or team. For instance, a sports fan is happier if she sees more other fans cheering for her team at the stadium; similarly, a voter enjoys a higher utility if her party obtains a larger vote share.

As we said, the purpose of this paper is to develop a model of group participation where peer pressure and an expressive externality coexist, and where peer pressure is generated as a norm put in place by the self-organizing group. A key conclusion that follows from our model is the possibility of "tipping." That is, whereas in peer pressure models the marginal cost of inducing additional participation is generally positive, when there is an expressive externality it may become negative, and beyond this tipping point it is optimal for the group to have all members to participate. Thus tipping cannot occur absent the externality. With that, whether or not it occurs is endogenous, and we give two applications. The first is to sporting events where we study how ticket pricing should be designed to exploit the externality. We show that if the externality is low then it may be desirable to keep ticket prices low to encourage participation; on the other hand if the externality is strong then the teams will optimally fill the stadium, and counter-intuitively may increase prices if the

⁴This is the famous "paradox of voting" (Downs, 1957).

⁵See, for example, Harsanyi (1980), Coate and Conlin (2004), Feddersen and Sandroni (2006) and Grillo (2022).

⁶Empirical evidence suggests that peer pressure indeed plays an important role in shaping voter turnout decisions. See, for instance, Grosser and Schram (2006), Gerber, Green, Larimer (2008) and DellaVigna, List, Malmendier and Rao (2016).

⁷On voting, recent and significant empirical evidence in favor of the long-standing literature on expressive voting can be found in Tyran and Wagner (2019), Pons and Tricaud (2018) and Rivas and Rockey (2021). Tyran and Wagner (2019) surveyed experimental evidence for expressive voting. Other surveys are Hamlin and Jennings (2011, 2019).

stadium capacity increases. Our second application is to voting where we demonstrate how voter turnout can change discontinuously although the underlying stakes are drawn from a continuous distribution. This leads to the prediction that turnout in elections may be twin-peaked. We conduct an empirical analysis to estimate an upper bound for the effect size of tipping, using aggregate voter turnout data from the US and UK. Our results suggest that tipping is relevant in UK although possibly not in US.

The core of the model is one of collective provision of incentives to participate. We know from the work of Ostrom (1990) and her successors how this can be achieved: groups can self-organize to overcome the free rider problem and provide public goods (such as participation) through peer monitoring and social punishments such as ostracism. Formal theories of peer enforcement originate in the work of Kandori (1992) on repeated games with many players and have been specialized to the study of organizations. The basic idea is that groups have norms consisting of a target behavior for the group members and individual penalties for failing to meet the target; these norms are endogenously chosen in order to advance group interests. Specifically the group designs a mechanism to promote group interests subject to incentive constraints for individual group members, and it provides incentives in the form of punishments for group members who fail to adhere to the norm.⁸

The paper speaks to the literature on sports ticket pricing and voter turnout. The literature about sports tickets studies factors that affect fans' attendance (Borland and MacDonald, 2003; Laverie and Arnett, 2000; Mastromartino and Zhang, 2020; Wakefield, 1995) and the revenue-maximizing ticketing strategy (Daniel, 1997; Drayer et al, 2012; Rodney, 2004). We point out how expressive externality – a practically important but largely overlooked factor in the literature – affects fans' attendance and how this can be exploited in the optimal pricing strategy. Our results imply that with strong expressive externality stadiums are always full under the optimal ticketing strategy. In the context of UK soccer for example, casual observation of full stadiums seems to confirm the presence of a high expressive externality.⁹ For voter turnout, our addition to the cited literature lies in the possibility that the turnout distribution can be twin peaked when there is an expressive externality. Our empirical analyses also suggest that such tipping effect seems to be stronger in UK than in US.

The sequel of the paper is organized as follows. The next section introduces the model. Sections 3 and 4 analyze the model and present our main tipping result. Section 5 examines the implication of tipping for optimal sports ticket pricing. Section 6 studies the implication

⁸See for example Levine and Modica (2016) and Dutta, Levine and Modica (2021).

⁹The average stadium attendance rates for clubs in the Premier League are close to 100% in almost all seasons. See https://www.transfermarkt.com/premier-league/besuchertzahlen/wettbewerb/GB1/plus/1?saison_id=2022.

of tipping for the distribution of turnout in large elections. Section 7 concludes.

2. The Model

The natural context for studying expression is that of a social network, a simple model of which is the following. A group k is composed of a continuum of members of size η_k (we shall consider cases in which the number of groups is either one or two). Each group member faces a participation decision: to root or not to root for the team; and if the group wins, which occurs with probability p_k , each group member has a utility v_k .¹⁰ The same goes for other participation decisions such as voting. The cost of participation of individual i is $c(y_i) = y_i - \beta$, where β is type-independent direct payoff from participation and the type y_i is uniformly distributed on $[0, 1]$.¹¹ Types are independent across individuals. In this paper we consider the addition of an expressive externality: you benefit from the participation of your peers, and the effect is proportional to the participation rate of your group. We denote by $\lambda \geq 0$ the strength of this externality. So if the fraction of participants in group k is ϕ_k then direct utility from participation of i is

$$\begin{cases} p_k v_k + \lambda \phi_k & \text{if } i \text{ does not participate} \\ p_k v_k + \lambda \phi_k + \beta - y_i & \text{if } i \text{ does participate.} \end{cases} \quad (2.1)$$

Note that without any intervention encouraging participation only those with types $y_i \leq \beta$ will choose to participate. We call these individuals *committed members*. Since y_i is uniformly distributed on $[0, 1]$ the fraction $\underline{\varphi}$ of committed members is then given by

$$\underline{\varphi} = \begin{cases} 0 & \text{if } \beta < 0 \\ \beta & \text{if } 0 \leq \beta \leq 1 \\ 1 & \text{if } \beta > 1. \end{cases} \quad (2.2)$$

Hereafter we will assume that $\beta < 1$ to avoid the uninteresting case in which all group members are committed; thus $\underline{\varphi} = \max\{0, \beta\}$.

Because of the expressive externality the group may have a collective interest in mobilizing higher participation rate than $\underline{\varphi}$. Throughout the paper we view each group k as a *self-organizing* group in the spirit of Levine and Modica (2016) and Levine and Mattozzi

¹⁰We will specify how p_k depends on participation rates of the own and rival groups in applications presented in Sections 4 to 6.

¹¹Here we let β represent various sources of direct benefit from participation, such as fulfillment of civic duty (Downs, 1957; Riker and Ordeshook, 1968; Blais, 2000) and expression (Brennan and Buchanan, 1984; Hamlin and Jennings, 2011).

(2020). More specifically, the group may self-organize to encourage the participation of non-committed members through peer monitoring and punishment. It does so by establishing a social norm whereby members with relatively low costs are expected to participate. Specifically, a *social norm* is defined as a threshold $\varphi_k \in [\underline{\varphi}, 1]$ for participation: those types with $y_i \leq \varphi_k$ are supposed to participate and those with $y_i > \varphi_k$ are not. If the social norm φ_k is followed, the expected fraction of the group that will participate is φ_k and in a large group we may assume that since we are averaging over many independent draws the realized participation ϕ_k is equal to the expected value φ_k .

The action of a member – whether she has participated or not – is observable to everyone, but for those who did not participate there is only a noisy signal of their type y_i . The signal is a binary signal $z_i \in \{0, 1\}$, where 0 means “*good, followed the social norm*” and 1 means “*bad, did not follow the social norm.*” Following Levine and Mattozzi (2020), we assume that if the social norm was violated (that is, the member did not participate but $y_i \leq \varphi_k$) the bad signal is generated with probability π_0 ; if i did not participate but $y_i > \varphi_k$ so that she did in fact follow the norm, there is nevertheless a chance π_1 of generating the bad signal. We assume that $\pi_1 \leq \pi_0$ so that the bad signal is more likely to be generated under a real violation of social norm.¹² Define $\pi \equiv \pi_1/\pi_0 \in [0, 1]$ as the likelihood ratio of bad signals. Higher π implies that signal is less informative about social norm violation. As we will see in the next section, the social cost of peer monitoring is proportional to π . If a member’s behavior generates a bad signal she suffers an endogenous punishment P_k that the group applies through some form of ostracism.

We model the behavior of the self-organizing group k as a mechanism design problem: to choose an incentive compatible pair of φ_k, P_k to maximize the *ex ante* per capita utility of group members. This per capita utility, which we derive below, is going to be the direct utility defined above minus the cost of implementing the chosen norm.

From the model formulation it is apparent that the direct utility, participation decision and the group self-organization problem are the same in the cases of sports teams rooting (where there is only one group, the home team fans), and voting in elections (where there are two opposite groups trying to win). The difference is that, as we will see, in the voting case the probability of winning depends on both groups’ behavior hence it is a bit more

¹²The idea is that in practice the actual participation of a citizen – whether one goes to the stadium, shows up in the poll station, attends a meeting – is relatively easy to observe by others (hence π_0 relatively high). On the other hand if a citizen does not participate the reason for her absence – for example, she just wants to shirk, or she is too busy or too sick to attend – is not directly observable to her peers (therefore π_1 relatively low). How well her peers can tell whether she violates the social norm or not depends on how close they are to her. This can be interpreted as a social trait and is captured by the ratio $\pi = \pi_1/\pi_0$ in our model. Perfect monitoring corresponds to the case $\pi = 0$, and a completely ineffective monitoring corresponds to the case $\pi = 1$.

involved than in the sports rooting case.

3. The Minimal Cost of Mobilization

As a first step we observe that maximizing utility requires that the cost of achieving a particular participation target φ_k is minimized. In other words, the self-organizing group must choose a punishment scheme P_k so that compliance with the social norm φ_k is incentive compatible. If everyone complies with the social norm bad signals are still generated with probability π_1 , so $\pi_1 P_k$ is a cost to the group of inducing compliance, which we call a *monitoring cost*; this is $M(\varphi_k) = \int_{\varphi_k}^1 \pi_1 P_k dy$. The *total cost* $C(\varphi_k)$ of implementing social norm φ_k is then the sum of monitoring cost and the *direct cost of participation* $T(\varphi_k) = \int_0^{\varphi_k} c(y) dy$. The former is exclusively driven by punishment misplaced on members with $y_i > \varphi_k$ whose abstentions are legitimate under the social norm. All these functions are defined for $\varphi_k \geq \underline{\varphi}$, because only these participation rates can be realized in the model. We now study the problem of minimizing the total cost for a given target φ_k .

Lemma 1. *Recall that $\pi = \pi_1/\pi_0$. The direct, monitoring, and total costs of inducing compliance with a social norm $\varphi_k \geq \underline{\varphi}$ are*

$$T(\varphi_k) = \varphi_k (\varphi_k - 2\beta) / 2, \quad \varphi_k \geq \underline{\varphi} \quad (3.1)$$

$$M(\varphi_k) = \begin{cases} 0 & \text{if } \varphi_k = \underline{\varphi} \\ \pi (1 - \varphi_k) (\varphi_k - \beta) & \text{if } \varphi_k > \underline{\varphi} \end{cases} \quad (3.2)$$

$$C(\varphi_k) = \begin{cases} \underline{\varphi} (\underline{\varphi} - 2\beta) / 2 & \text{if } \varphi_k = \underline{\varphi} \\ (\frac{1}{2} - \pi) \varphi_k^2 - \pi\beta + [\pi - (1 - \pi)\beta] \varphi_k & \text{if } \varphi_k > \underline{\varphi} \end{cases} \quad (3.3)$$

The optimal level of punishment is $P_k = (\varphi_k - \beta) / \pi_0$.

Proof. By definition the direct cost of participation above the committed level $\underline{\varphi}$ is $T(\varphi_k) = \int_0^{\varphi_k} c(y) dy$ for $\varphi_k \geq \underline{\varphi}$, and direct computation gives

$$T(\varphi_k) = \int_0^{\varphi_k} c(y) dy = \varphi_k^2 / 2 - \beta \varphi_k = \varphi_k (\varphi_k - 2\beta) / 2, \quad \varphi_k \geq \underline{\varphi}$$

Next we derive the monitoring cost. The incentive constraint is that members with $y_i \leq \varphi_k$ should be willing to participate, that is $y_i - \beta \leq \pi_0 P_k$ for all $y_i \leq \varphi_k$ - therefore it must be $\pi_0 P_k \geq \varphi_k - \beta$; and members with $y_i > \varphi_k$ should not, that is $\pi_1 P_k \leq y_i - \beta$ for all $y_i > \varphi_k$ or equivalently $\pi_1 P_k \leq \varphi_k - \beta$. Because $\pi_1 \leq \pi_0$ and minimization of cost implies that the constraint should bind, we obtain that $P_k = (\varphi_k - \beta) / \pi_0$. Notice that without monitoring (that is, no punishment exerted to non-participants) the participation rate is

lowest and equal to $\underline{\varphi}$. Therefore, $M(\underline{\varphi}) = 0$. For $\varphi_k > \underline{\varphi}$, recalling that for these values $P_k = (\varphi_k - \beta) / \pi_0$, the monitoring cost is

$$M(\varphi_k) = \int_{\varphi_k}^1 \pi_1 P_k dy = \frac{\pi_1}{\pi_0} \int_{\varphi_k}^1 (\varphi_k - \beta) dy = \pi (1 - \varphi_k) (\varphi_k - \beta)$$

Taking these results together, we obtain (3.2). Hence the total cost $C(\varphi_k) \equiv T(\varphi_k) + M(\varphi_k)$ of inducing any social norm $\varphi_k \geq \underline{\varphi}$ is easily verified to be given by (3.3). \square

The direct cost $T(\varphi_k)$ is strictly convex in φ_k , while the monitoring cost $M(\varphi_k)$ is concave in φ_k for all $\pi > 0$ and it is proportional to π , the noisiness of the social signal z_i about norm violation. Importantly, the total cost $C(\cdot)$ is strictly convex (resp. concave) for $\pi < 1/2$ (resp. $\pi > 1/2$). This implies that the marginal cost of mobilization is increasing (decreasing) in participation rate as the difficulty of monitoring π is below (above) $1/2$.

To simplify the analysis we assume $\pi = 1/2$, so that the net marginal cost of mobilizing participation is constant, but we check that the tipping result underpinning the analysis is robust to relaxing this assumption. With $\pi = 1/2$ it follows directly from (3.3) that the cost function $C(\varphi_k)$ is given by the following;

Corollary 1. *Suppose $\pi = 1/2$. Then*

$$C(\varphi_k) = \begin{cases} \underline{\varphi} (\underline{\varphi} - 2\beta) / 2 & \text{if } \varphi_k = \underline{\varphi} \\ \frac{1-\beta}{2} \varphi_k - \frac{\beta}{2} & \text{if } \varphi_k > \underline{\varphi} \end{cases}. \quad (3.4)$$

4. Main Result: Tipping

In this section and the next we assume that there is only one group k and normalize its size η_k to one. In this context we interpret participation as an expression of support for the group: for instance cheering for a sports team at the stadium or a pop star at a concert.¹³ Our goal is to understand how the group's optimal social norm φ_k depends upon the strength of the externality λ . In particular we will establish that there is tipping in the sense that there is a critical value λ^* for which participation jumps discontinuously as λ exceeds this threshold. Such tipping can also be triggered by continuous changes of other model parameters when $\lambda > 0$ is sufficiently large. It is these tipping phenomena that is the main topic of the paper. All proofs for this section are relegated to Appendix A.

To present the tipping result in the most transparent way, we first assume for simplicity that $\pi = 1/2$ (as indicated before) and $v_k = 0$, that is, the only benefit to the group from

¹³In Section 6 we study the case with two groups, whereby two political parties strategically mobilize their voters in an electoral competition.

participation are the individual benefits from expression. As we shall see later, our tipping result is robust when these assumptions are relaxed. Observe that $\phi_k = \varphi_k$ must hold to be in compliance with any social norm $\varphi_k \geq \underline{\varphi}$. Lemma 2 characterizes the per capita utility for each member in group k as a function of social norm φ_k under our simplifying assumptions.

Lemma 2. *Assume $\pi = 1/2$, $v_k = 0$ and let*

$$\xi \equiv \frac{1 - \beta}{2} - \lambda. \quad (4.1)$$

Then the per capita group utility is equal to

$$\mathcal{U}_k(\varphi_k) = -\xi\varphi_k + \frac{\beta}{2} \cdot \mathbf{1}\{\varphi_k > 0\} \quad \text{for } \varphi_k \in [\underline{\varphi}, 1] \quad (4.2)$$

Here $\mathbf{1}\{\cdot\}$ is the indicator function. Note that ξ represents the (constant) marginal cost of inducing participation rate φ_k net of the benefit from expressive externality. Assume $\beta < 1$ so that the fraction of committed members $\underline{\varphi}$ is less than one (cf. (2.2)). Then, without the expressive externality (i.e., $\lambda = 0$) this marginal cost ξ is strictly positive and the optimal participation is unambiguously $\underline{\varphi}$. Thus without the externality we have a pretty standard model – self-organization never creates a discontinuity in participation rate: only the committed members participate and the fraction of these members is a continuous function of model parameters (Levine and Modica, 2016; Levine and Mattozzi, 2020).

Now assume $\lambda > 0$. Recall that $\underline{\varphi} = \max\{0, \beta\}$, and let

$$\lambda^* \equiv \begin{cases} \frac{1-\beta}{2} & \text{if } \beta \geq 0 \\ \frac{1-\beta}{2} - \frac{\beta}{2} & \text{if } \beta < 0 \end{cases} \quad (4.3)$$

With φ_k^* denoting the optimal social norm for group k , we have

Proposition 1 (Tipping). *Suppose $\pi = 1/2$ and $v_k = 0$. Then the optimal social norm of participation increases discontinuously in λ at threshold λ^* :*

$$\varphi_k^* = \begin{cases} \underline{\varphi} & \text{if } \lambda < \lambda^* \\ 1 & \text{if } \lambda > \lambda^* \end{cases} \quad (4.4)$$

The tipping threshold λ^ is decreasing in β .*

To see the intuition, recall that $\xi \equiv (1 - \beta)/2 - \lambda$ is the constant marginal cost of increasing participation net of the benefit of externality. The key point is that this is positive for sufficiently low externality λ but becomes negative when λ is large. If $\beta \geq 0$ –

so that the lowest cost member with $y_i = 0$ weakly prefers to participate – it follows from (4.2) that $\mathcal{U}_k(\cdot)$ is a linear function on $[\underline{\varphi}, 1]$. In this case, $\xi < 0$ if and only if $\lambda > \lambda^*$ and it is this switch from positive to negative marginal cost at λ^* that triggers tipping. If instead $\beta < 0$ – so that $\underline{\varphi} = 0$ and participation incurs strictly positive costs for all member types – it follows from (4.3) that the tipping threshold λ^* is higher so that a higher level of externality is required to trigger tipping. This is because, by (4.2), $\mathcal{U}_k(0) = 0$ and $\lim_{\varphi_k \downarrow 0} \mathcal{U}_k(\varphi_k) = \beta/2 < 0$ when $\beta < 0$. That is, $\mathcal{U}_k(\varphi_k)$ has a downward jump at zero. The size of this downward jump is exactly the monitoring cost of inducing the lowest type to participate.¹⁴ It is this “start-up cost” of mobilizing the lowest cost member that makes inducing tipping less profitable for the group, and hence a higher level of externality λ is required to trigger tipping.

The tipping result extends easily to other model parameters when $\lambda > 0$. For example, by (3.2) the marginal cost ξ is strictly decreasing in β . Then, following the similar logic of tipping for λ , there also exists corresponding tipping threshold β^* – such that participation can jump up discontinuously as β crosses β^* from below – provided that $\lambda > 0$ is sufficiently large.¹⁵ Such discontinuity, as explained above, is impossible absent the expressive externality. It is hence the possibility that participation can vary discontinuously with parameters that features the essential implication of expressive externality. In the next two sections we will consider other variants of tipping results induced by model parameters other than λ . Despite their seemingly differences in presentation, readers should bear in mind that the mechanisms that these parameters trigger tipping are exactly the same.

4.1. Robustness

In this section we relax the assumptions $\pi = 1/2$ and $v_k = 0$ made previously and show that the tipping result is robust. With only one group we may assume that the probability of success p_k is a non-decreasing function of the participation rate φ_k ; for simplicity we take it to be linear, with slope $\psi \geq 0$. The result is the following:

Proposition 2 (Robustness of Tipping). *Suppose $v_k > 0$ and $p_k = p_k(\varphi_k)$ is a linear function with slope $\psi \geq 0$. Define $\lambda^{**} \equiv \lambda^* - \psi v_k$. Then the following holds:*

1. *If $\pi > 1/2$, then $\varphi_k^* = \underline{\varphi}$ for $\lambda < \lambda^{**}$ and $\varphi_k^* = 1$ for $\lambda > \lambda^{**}$. That is, the tipping result in Proposition 1 still applies with tipping point λ^* shifted to λ^{**} .*

¹⁴More precisely, this jump equals the discontinuous increment of the total cost function $C(\cdot)$ at $\varphi_k = 0$ when $\beta < 0$ (cf. 3.4). Since the direct cost $T(\cdot)$ is continuous and equals zero at $\varphi_k = 0$, the jump in total cost at $\varphi_k = 0$ is entirely driven by the monitoring cost.

¹⁵This is because only for sufficiently large $\lambda > 0$ it is possible that $\xi < 0$ and $\underline{\varphi} < 1$ – which are necessary for tipping – can simultaneously hold.

2. If $\pi = 1/2 - \epsilon$ for $0 < \epsilon < 1/2$ and $\beta \geq 0$, then¹⁶

$$\varphi_k^* = \begin{cases} \underline{\varphi} & \text{if } \lambda \leq \lambda^{**} - 2\epsilon\lambda^* \\ \frac{1}{2\epsilon}(\lambda - \lambda^{**}) + \frac{1+\underline{\varphi}}{2} & \text{if } \lambda \in (\lambda^{**} - 2\epsilon\lambda^*, \lambda^{**} + 2\epsilon\lambda^*) \\ 1 & \text{if } \lambda \geq \lambda^{**} + 2\epsilon\lambda^* \end{cases} \quad (4.5)$$

Part (1) of Proposition 2 says that when $\pi > 1/2$ and $p'_k(\cdot) = \psi \geq 0$ the basic tipping result is essentially unchanged, with just that the tipping point is now shifted downwards by ψv_k . Therefore, the required threshold of λ to trigger tipping becomes lower when $\psi > 0$, that is, participation has a positive effect in public good provision. This is because, like the expressive externality λ , a positive ψ features a standard *productive externality* that increases the marginal benefit of mobilizing participation. This additional benefit from public good provision makes inducing tipping more likely to be optimal for the group. Part (2) of Proposition 2 says that for $\pi = 1/2 - \epsilon$ and ϵ small the result is that φ_k^* as a function of λ increases steeply from $\underline{\varphi}$ to 1 in the interval $(\lambda^{**} - 2\epsilon\lambda^*, \lambda^{**} + 2\epsilon\lambda^*)$ whose width tends to zero. We therefore conclude that tipping is robust when $\pi > 1/2$, and it is a good approximation if $\pi = 1/2 - \epsilon$ for ϵ sufficiently small.

We observe that Proposition 2 implies that to trigger tipping π must be sufficiently away from 0 so that the monitoring cost is substantial. This is because, as explained in Section 3, the monitoring cost changes the curvature of the total cost function $C(\cdot)$ and hence the marginal cost of participation mobilization. Only when π is sufficiently high the marginal cost of mobilization can be decreasing, and this provides the crucial incentive for the group to engage in costly efforts to boost the highest participation.

5. Implications of Tipping for Sports Ticket Pricing

In this section we apply the tipping result to study the problem faced by a sports team whose fans are a self-organizing group.¹⁷ Our goal is to study the implications the possibility of triggering tipping brings to the optimal ticket pricing of a sports team. Here participation takes the form of attending a match, and the sports team charges a price r for attending. We assume in addition that the sports stadium has a maximum capacity and can accommodate only a fraction $Q \leq 1$ of its fans. Subject to the capacity constraint the

¹⁶For $\beta < 0$ and $\pi < 1/2$ the result is qualitatively the same, but the statement is more involved so we deal with it in Appendix A, where this proposition is proven.

¹⁷The importance of social influence on sports attendance has long been recognized in the literature (Wakefield, 1995). Indeed, it is very common for sports teams to have supporters' groups, which are organizations that represent their fans. These supporter's groups often engage in activities that encourage fans' participation. See, for instance, https://en.wikipedia.org/wiki/Supporters%27_group.

costs of the team are entirely sunk so it wishes to choose r to maximize revenue.

For clarity of exposition, we continue to assume $\pi = 1/2$. Following the spirit of Proposition 2 in the previous section, we assume $v_k > 0$ and $p_k(\phi_k)$ is a linear function with slope $\psi \geq 0$. This is a parsimonious way to model the fact that fans of a sports team typically enjoy a public benefit when their home team wins, and that fans' attendance at stadium may have a positive effect on the home team's winning probability.¹⁸ Since the sports team charges a fee r for attendance, the constant β in $c(y_i) = y_i - \beta$ becomes $\beta = \beta_0 - r$. We assume $\beta_0 > 0$ because otherwise attendance would be zero even when tickets are free.

As a benchmark, we first derive the optimal solution when the fans' group does not self-organize. In this case the sports team only sells to committed fans whose fraction equals $\underline{\varphi}$. By (2.2) and the fact that $\beta = \beta_0 - r$, we have

$$\underline{\varphi} = \begin{cases} 0 & \text{if } \beta_0 - r < 0 \\ \beta_0 - r & \text{if } 0 \leq \beta_0 - r \leq 1 \\ 1 & \text{if } \beta_0 - r > 1. \end{cases} \quad (5.1)$$

Then we have a standard monopoly pricing problem:

$$\max_r \{r \cdot \min\{\beta_0 - r, Q\}\}$$

where r is the ticket price the sports team charges and clearly $\beta_0 - r \geq 0$ must hold for any optimal r . The maximum of $r(\beta_0 - r)$ is $r^M = \beta_0/2$ giving $\underline{\varphi} = \beta_0/2 > 0$. If this $\underline{\varphi}$ is less than the stadium capacity Q then $\beta_0/2$ is the optimal price and the induced attendance. If $\underline{\varphi}$ is greater than Q then r^M should be chosen so that $\beta_0 - r^M = Q$, or equivalently $r^M = \beta_0 - Q$. Therefore the sports team's optimal strategy is to sell at price

$$r^M(Q) = \max\{\beta_0 - Q, \beta_0/2\} = \begin{cases} \beta_0 - Q & \text{if } Q < \beta_0/2 \\ \beta_0/2 & \text{if } Q \geq \beta_0/2 \end{cases}. \quad (5.2)$$

Observe that the optimal price $r^M(Q)$ without group self-organization shares two features that are common in standard monopoly pricing models. First, $r^M(Q)$ is continuous and weakly decreasing in capacity Q (strictly so if this capacity is binding). Second, there is no rationing in the following sense; if the price is fixed at some level $r^M(Q)$, the attendance

¹⁸The empirical evidence on how fans' attendance affects the home team's performance is mixed. For example, Cross and Uhrig (2023) and Smith and Groetzingler (2010) find positive effects of fan attendance on home team advantage in top European soccer leagues and the Major League Baseball (MLB), respectively. In contrast, Belchior (2020) and Böheim, Grübl and Lackner (2019) find no (or even negative) effects of fan attendance in Brazilian football and the National Basketball Association (NBA).

rate (the demand for tickets) is unaffected when capacity Q is expanded. Put differently, there can never be excessive demand over capacity Q . We will see that both features can be overturned when the possibility of tipping is taken into account for pricing.

Observe that the fraction of Q demanded as a function of price is $\underline{\varphi}$. So if $r = \beta_0 - Q$ then $\underline{\varphi} = Q$ - so there capacity is fully used up to $Q = \beta_0/2$. If $Q > \beta_0/2$ then $\underline{\varphi} = \beta_0/2 < Q$, so there is slack capacity.

Now suppose the fans' group can boost attendance through imposing a social norm. In Appendix B we show that, under $\pi = 1/2$ and given ticket price r , the marginal cost of mobilizing participation is

$$\xi \equiv \frac{1}{2} - \frac{\beta_0 - r}{2} - \lambda - \psi v_k. \quad (5.3)$$

Therefore, the sports team in effect controls the marginal cost of mobilization by choosing ticket price r . Recall from Section 4 that a necessary condition to trigger tipping is that the marginal cost ξ is non-positive. Beyond this point the optimal participation rate is Q (the highest fraction of fans that fits into the stadium). By (5.3), ξ is strictly increasing in r and $\xi \leq 0$ if and only if

$$r \leq r^I \equiv \beta_0 + 2(\lambda + \psi v_k) - 1. \quad (5.4)$$

The ticket price r thus must be sufficiently low to trigger tipping.

The question for the sports team is then: should it set a price above the tipping price and sell just to committed fans $\underline{\varphi}$ or set the tipping price and fill the stadium? To answer this question it is important to realize that the price ceiling r^I is nondecreasing in Q and strictly increasing in λ . So, unlike the standard monopoly case, under tipping the optimal price will not decrease in stadium size Q , and inducing tipping is more profitable as Q increases. Therefore, inducing tipping is optimal if and only if Q exceeds some threshold Q_λ^* that is decreasing in λ . This yields interesting relationship between price and capacity at different levels of λ . This is stated in Proposition 3 and illustrated by Figure 5.1.

Proposition 3. *Assume $\beta_0/2 \in (0, 1)$ and define*

$$\underline{\lambda} \equiv \frac{1}{2} \left(1 - \frac{\beta_0}{2} \right)^2 - \psi v_k, \quad \tilde{\lambda} \equiv \frac{1}{2} \left(1 - \frac{\beta_0}{2} \right) - \psi v_k \quad \text{and} \quad \bar{\lambda} \equiv \frac{1}{2h} - \psi v_k .$$

The optimal price $r^(Q; \lambda)$ as a function of stadium capacity Q is given as follows:*

1. *If $\lambda \leq \underline{\lambda}$ then triggering tipping is never optimal so the optimal price $r^*(Q; \lambda) = r^M(Q)$ for all $Q \in [0, 1]$.*
2. *If $\underline{\lambda} < \lambda \leq \bar{\lambda}$ then there is a cutoff Q_λ^* (strictly decreasing in λ) such that triggering*

tipping is optimal if and only if $Q \geq Q_\lambda^*$.¹⁹ The optimal price is given by

$$r^*(Q; \lambda) = \begin{cases} r^M(Q) & \text{if } Q < Q_\lambda^* \\ \beta_0 + 2(\lambda + \psi v_k) - 1 & \text{if } Q \geq Q_\lambda^* \end{cases}.$$

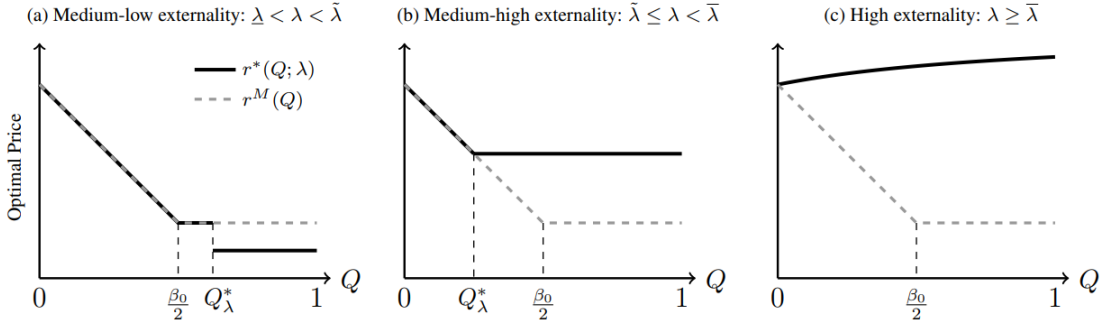
$r^*(Q)$ is weakly decreasing in Q , and it has a discontinuous downward jump at Q_λ^* if $\lambda < \bar{\lambda}$ (Figure 5.1(a)) and is continuous at Q_λ^* for $\lambda \geq \bar{\lambda}$ (Figure 5.1(b)).

3. If $\lambda > \bar{\lambda}$ then triggering tipping is always optimal independent of capacity Q . The optimal price is given by

$$r^*(Q; \lambda) = \beta_0 + (2(\lambda + \psi v_k) - 1) \frac{Q}{1 + Q}$$

and it is continuous and strictly increasing in Q (Figure 5.1(c)).

Figure 5.1: Relationship between the optimal ticket price $r^*(Q; \lambda)$ and stadium capacity Q when $\lambda > \underline{\lambda}$.



Note: When tipping is impossible the optimal price (gray dashed line) is $r^M(Q)$ in (5.2), which is the standard monopoly case. When tipping is possible (black line) Q_λ^* is the critical point for regime switching: it is optimal to charge the tipping price if and only if $Q \geq Q_\lambda^*$. This cutoff Q_λ^* is strictly decreasing in λ on $[\underline{\lambda}, \bar{\lambda}]$ and $Q_\lambda^* = \beta_0/2$ at $\lambda = \bar{\lambda}$.

Proposition 3 suggests that when the expressive externality is too low ($\lambda \leq \underline{\lambda}$) it is never optimal to induce tipping and hence the best strategy for the sports team is to charge the standard monopoly price $r^M(Q)$, which is continuous and strictly decreasing in Q . In sharp contrast however, when the externality is very high ($\lambda > \bar{\lambda}$) it is optimal to induce tipping (and fill the stadium) independently of Q ; in this case the price is strictly increasing in stadium size. This is because a larger stadium size allows each fan to enjoy a higher payoff from more participation of others, thanks to the strong expressive externality. This then allows the sports team to charge higher prices without discourage fans to collectively induce

¹⁹The precise expression of Q_λ^* is derived in Appendix B. It turns out that Q_λ^* is continuous and strictly decreasing from 1 to 0 as λ increases from $\underline{\lambda}$ to $\bar{\lambda}$, and $Q_\lambda^* = \beta_0/2$.

maximal participation.

For intermediate $\lambda \in (\underline{\lambda}, \bar{\lambda})$ it turns out that, interestingly, there is an interior $Q_\lambda^* \in (0, 1)$ such that charging the tipping price is optimal if and only if $Q \geq Q_\lambda^*$. This Q_λ^* is strictly decreasing in λ , consistent with the intuition that tipping is more likely to be optimal with greater expressive externality. This regime switch triggers subtle patterns for how λ affects the relationship between optimal price and capacity. When $\lambda > \tilde{\lambda}$ under the optimal ticket price the stadium is always filled for all capacity levels $Q \in [0, 1]$. The reason is that for $Q < Q_\lambda^*$ the price is equal to monopoly price, but in this region $Q < \beta_0/2$ so stadium is full anyway. Moreover, price is never lower than monopoly price without tipping. When $\lambda < \tilde{\lambda}$, in the range $Q \in [\beta_0/2, Q_\lambda^*]$ it is not worthwhile inducing tipping and there is slack capacity since the optimal price is the monopoly price, which in this range calls for less than full capacity. At $Q = Q_\lambda^*$ tipping is induced and to do so a discrete price drop is need to provide the group incentives to mobilize fans. The price is no higher than monopoly pricing without tipping, and there could possibly be a discontinuous jump downwards.

Finally, the optimal price can respond to externalities λ and ψ , while this is not the case when tipping is impossible. This is because it makes sense for the optimal price to respond to externalities only when the sports team expects the group's organization efforts to be strategic and effective; after all, individual decisions never take externalities into account. As ψ increases it becomes more likely to be optimal for the team to induce tipping, and the tipping price charged is increasing in ψ .²⁰ One interesting way to view this result is to interpret ψ as fans' belief about the effectiveness of their attendance in improving the home team's performance. Such belief ψ can be affected by persuasive narratives that emphasize or exaggerate fans' importance. Our result then implies that the sports team may have strong incentives to increase ψ by costly investing in such persuasion even when fans' attendance has objectively little effect on team performance.²¹ This is because a higher belief ψ shared by fans will allow the team to earn more profits through triggering tipping. We believe that this is an interesting possibility result that worth empirical investigations.

To sum up, our results illustrate how the sports team's optimal pricing should exploit fans' self-organization behavior. These insights convey to other entertainments such as selling tickets for concerts and super stars, etc.

²⁰In reality the objective of a sports team may be a combination of both profit maximization and improving team performance (say for future reputation and potential benefits; like upgrade to higher level). In this case the standard monopoly price without tipping $r^M(Q)$ can be continuous and decreasing in ψ . Under tipping, however, the optimal price must be either weakly increasing or has a discontinuous drop as ψ increases continuously. These comparative static predictions are qualitatively very different.

²¹When tipping is impossible this cannot happen; fans' individual participation decision does not take externality ψ into account and hence spreading such narratives will not influence their behavior.

Do Ticket Prices Rise When Larger Stadiums Are Built?

From an empirical point of view it would be interesting to know which panel of Figure 5.1 is relevant in a particular situation. Of course many factors influence the dependence of prices on stadium capacity. For an example, in the US it seems that new, larger stadiums indeed bring higher ticket prices. An article²² indicates that for three new US football facilities prices in the new stadiums were 26% higher than in the old. The capacity of the new and old stadiums is reported in Wikipedia: capacity increased by an average of 15%.

6. Tipping result for two groups and application to voter turnout

In this section we extend our tipping result for two groups competing in all-pay auction contests (e.g., Levine and Mattozzi (2020)). To fix ideas, we interpret the two groups as political parties $k \in \{L, S\}$ who compete in an election. The relative size of the two parties is $\eta_L > \eta_S > 0$ with $\eta_L + \eta_S \leq 1$, where η_k is the fraction of people who would vote for $k = L, S$ with certainty *if* they decided to vote.²³ The party that sends the most voters to the polls wins the election and receives a total prize of size V ; the per capita value of the public good for each party k to win the election is thus $v_k = V/\eta_k$. We make the following assumption.

Assumption 1. $\pi = 1/2$, $\beta \in (0, 1)$, $\lambda = \kappa V$ and $\kappa > 0$.

Assumption $\beta \in (0, 1)$ ensures that the fraction of committed voters is $\varphi = \beta \in (0, 1)$ for both parties (cf. (2.2)). Following Levine and Mattozzi (2020), we define a *bid* b_k by group k as the number of voters mobilized to turnout by party k , that is $b_k = \eta_k \varphi_k$. For each party $k = L, S$, the set of feasible bids is given by $[\beta \eta_k, \eta_k]$. Let $\Pi_k(b_k, b_{-k})$ denote the winning probability of party k as a function of bids b_k and b_{-k} submitted by both parties. We also assume that the large party L wins the election in case of a tie.²⁴ Therefore

$$\Pi_L(b_L, b_S) = \begin{cases} 1 & \text{if } b_L \geq b_S \\ 0 & \text{if } b_L < b_S \end{cases} \quad \Pi_S(b_S, b_L) = 1 - \Pi_L(b_L, b_S).$$

²²<https://profootballtalk.nbcsports.com/2012/05/28/new-stadiums-are-resulting-in-dramatically-increased-ticket-prices/>

²³To facilitate our empirical exercise presented in subsection 6.1, we will introduce a strategically neutral group of “civic voters” with size $\eta_c = 1 - \eta_L - \eta_S$ that mechanically split their votes equally to both parties and do not affect the outcome of the election.

²⁴This is to simplify equilibrium analyses and guarantee that equilibria always exist. See footnote 8 of Levine and Mattozzi (2020) for a more detailed discussion.

Let $\bar{V} \equiv (1 - \beta) / (2\kappa)$. Our main result is that tipping takes place as V crosses \bar{V} from below: the details are in Appendix C.²⁵

Proposition 4. *Suppose Assumption 1 holds and $\eta_S > \beta\eta_L$. For $V > \bar{V}$ the aggregate bid by the two parties is $\bar{b} = \eta_S + \eta_L$; for $V < \bar{V}$ close to \bar{V} aggregate bid is approximately $\underline{b} = (1 + \beta)\eta_S$.*

For $V > \bar{V}$ all voters in both parties turn out and the aggregate bid $b = b_L + b_S$ by the two parties is $\bar{b} = \eta_S + \eta_L$, independently of the probability of winning. For $V < \bar{V}$ we have to study the game between the two parties because the probability of winning depends on bids. The analysis in Appendix C shows that for V close to \bar{V} – which is the relevant case in general elections, where stakes are high – in equilibrium to a good approximation the large party bids η_S and the small party bids $\beta\eta_S$, so the aggregate bid is approximately $\underline{b} = (1 + \beta)\eta_S$. The main implication of this result is that the distribution of voter turnout is twin-peaked due to tipping: even if V has a continuous single-peaked distribution, as V crosses \bar{V} we should observe a discontinuous upward jump in turnout. This implies that the distribution of voter turnout shall follow a distribution with more than one peak. We examine this empirically, and estimate an upper bound about the extent to which such tipping can matter.

6.1. Effect sizes of tipping in US and UK

Having established the possibility of tipping for voter turnout, a natural question to ask is to what extent tipping is relevant in explaining variations in voter turnout in practice. This is what we explore next, using aggregate turnout data from both the US and UK. We focus on general elections and presidential elections because the stakes are most salient, and parties' incentives to mobilize turnout are also the strongest (Shachar and Nalebuff, 1999; Levine and Mattozzi, 2020).²⁶ Our goal is to estimate an upper bound for the effects of tipping. To facilitate this estimation, we first slightly perturb the model above and show that under reasonable simplification assumptions the distribution of voter turnout can be given by a mixture of two normal distributions with different peaks; the parameters of this model can be estimated using the maximum likelihood approach. We then interpret the estimation results in light of our tipping model; the rationale is that we hypothetically attribute all the estimated effects to tipping and this reasonably gives an upper bound.

²⁵While in Proposition 4 tipping is triggered by V , it should be clear that a similar tipping result can also be triggered by other parameters such as β and κ .

²⁶See Shachar and Nalebuff (1999) and Section V.B of Levine and Mattozzi (2020) for discussions of related empirical evidence.

6.1.1. *The Bernoulli-Normal mixture model for voter turnout*

To account for the fact that the actual voter turnout is certainly not Bernoulli and it in general takes continuous and interior values (that is, away from 1 and 0), we introduce a third group of civic voters of size $\eta_c = 1 - \eta_L - \eta_S$, who split votes equally between the two parties (so that the strategic aspects of voting are unchanged). We further assume that each civic voter i faces a participation cost $y_i - \beta_c$, where $y_i \sim U(0, 1)$. The fraction of committed voters from this group is determined by $y_i - \beta_c \leq 0$ so it is β_c . We assume that β_c is normally distributed with mean μ_c and standard deviation σ_c . Essentially, what civic voters do is to add additional terms in parties' bids: when parties bid b_L and b_S , the actual fraction of voters that cast votes for them are:

$$\tau_S = b_S + \beta_c \eta_c / 2 \quad \tau_L = b_L + \beta_c \eta_c / 2$$

where $1/2$ shows up because civic voters are divided equally to parties. The aggregate turnout is then

$$\tau := \tau_L + \tau_S = b + \beta_c \eta_c \tag{6.1}$$

where $\beta_c \sim \mathcal{N}(\mu_c, \sigma_c)$ and b is Bernoulli with probability Q_1 of \underline{b} (the probability that $V < \bar{V}$) and $1 - Q_1$ of \bar{b} (probability of $V > \bar{V}$). Equivalently, τ is a mixture of two normal distributions both with standard deviation $\sigma = \eta_c \sigma_c$, and one with mean $\mu_1 = \eta_c \mu_c + \underline{b}$ and the other with mean $\mu_1 + g = \mu_c \eta_c + \bar{b}$.²⁷ Therefore, the gap $g = \bar{b} - \underline{b}$ captures the effect size of tipping on voter turnout. We call this model a Bernoulli-Normal mixture.

6.1.2. *Estimating tipping effects in US and UK*

We now estimate the tipping effect using the Bernoulli-Normal mixture model derived above. To do so we gathered aggregate turnout data from US presidential elections (1920-2020) and UK general elections (1918-2019) beginning with the first election in which women were permitted to vote.

In our model turnout τ_t in each period t is a mixture of two normal distributions $\mathcal{N}(\mu_1, \sigma^2)$ and $\mathcal{N}(\mu_1 + g, \sigma^2)$. The probability that τ_t is drawn from $\mathcal{N}(\mu_1, \sigma^2)$ equals $Q_1 \in (0, 1)$. Without loss of generality, we assume $g > 0$ to ensure identification. Let $\vartheta = (Q_1, \mu_1, g, \sigma)$ be the vector of parameters. A crucial fact, as we show in Appendix D, is that voter turnout has strong positive serial correlation and is a stationary process. With this positive serial correlation the exact likelihood function is not tractable as it requires us to compute for each set of parameters a likelihood for each possible sequence of Bernoulli of which there are many. Instead we implement a “partial” maximum likelihood

²⁷While the actual turnout is bounded between zero and one, a finite normal mixture model may well approximate it when the standard deviations are sufficiently small.

approach as described in Levine (1983). Here we obtain consistent estimates by maximizing the product of the stationary density functions, that is, proceeding “as if” the observations were independently drawn from its stationary distribution.²⁸ The standard errors are then computed using both contemporaneous and lagged information matrices. The stationary density function is then

$$f(\tau_t|\vartheta) = Q_1\phi(\tau_t; \mu_1, \sigma^2) + (1 - Q_1)\phi(\tau_t; \mu_1 + g, \sigma^2)$$

where $\phi(x; \mu, \sigma^2) = (1/\sqrt{2\pi\sigma^2}) \exp(-(\tau_t - \mu)^2/2\sigma^2)$ denotes the density function for $\mathcal{N}(\mu, \sigma^2)$. Then, given the time series of turnout $\boldsymbol{\tau} := \{\tau_t\}_{t=1}^T$, the partial log-likelihood function is

$$\begin{aligned} \mathcal{L}(\vartheta; \boldsymbol{\tau}) &= \sum_{t=1}^T \log f(\tau_t|\vartheta) \\ &= \sum_{t=1}^T \log \left(Q_1 e^{-\frac{(\tau_t - \mu_1)^2}{2\sigma^2}} + (1 - Q_1) e^{-\frac{(\tau_t - \mu_1 - g)^2}{2\sigma^2}} \right) - \frac{T}{2} \log 2\pi\sigma^2 \end{aligned} \quad (6.2)$$

We estimate $\vartheta = (Q_1, \mu_1, g, \sigma)$ using the maximum likelihood approach. The estimation results are presented in Table 1.

Table 1: ML Estimation results for US and UK

Parameters	U.S. presidential elections (1920-2020)	U.K. general elections (1918-2019)
\hat{g}	0.066 (0.014)	0.127 (0.034)
\hat{Q}_1	0.626 (0.233)	0.177 (0.159)
$\hat{\sigma}_B$	0.032 (0.006)	0.049 (0.006)
$\hat{\mu}_1$	0.529 (0.009)	0.622 (0.031)
$\hat{\sigma}$	0.027 (0.005)	0.041 (0.006)
Partial Log-likelihood	46.683	39.788
Observations	26	28

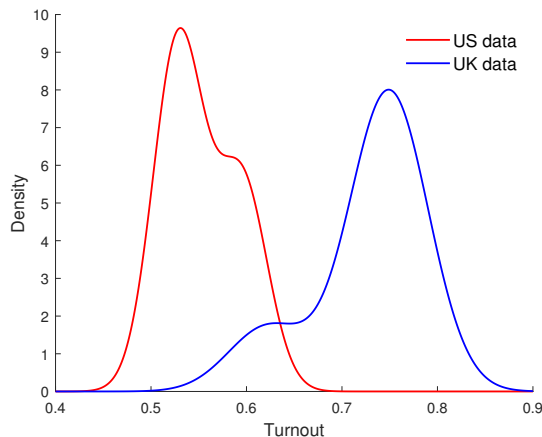
Note: In this table $\hat{\sigma}_B \equiv \hat{g}\sqrt{\hat{Q}_1(1 - \hat{Q}_1)}$ is the estimated standard deviation of the Bernoulli component in the mixture model. Robust standard errors are reported in parentheses and they are computed following the method in Levine (1983) with lag $k = 4$. The choice $k = 4$ is made based on a tradeoff between bias and precision of estimates. As Table 2 in Appendix D shows, the serial correlation is around 0.5. It can be checked that for an AR(1) with a coefficient of 0.5 the contribution of lags after 4 to the stationary standard error is less than 1/10th of a percent. The choice of $k = 4$ also leaves us with 21 to 23 observations to use in the estimation.

²⁸That is, the estimated $\vartheta = (Q_1, \mu_1, g, \sigma)$ are interpreted as parameters of the stationary distribution of the time series data for voter turnout.

We are mostly interested in the size of the gap \hat{g} and the standard deviation of the Bernoulli component $\hat{\sigma}_B$. Both are large in economic terms: the estimated \hat{g} 's indicate a 6.6% increment to turnout due to tipping in the US and 12.7% in the UK. The effect size of tipping in UK is thus about the double of the size of tipping in US. Also, bearing in mind that at $Q_1 = 1/2$ the standard error is half the gap, we find similarly large standard deviations $\hat{\sigma}_B$ of the Bernoulli component: 3.2% in the US and 4.9% in the UK. More strikingly, in both US and UK the estimated $\hat{\sigma}_B$'s are larger than the estimated standard deviations $\hat{\sigma}$'s of the normal component. This suggests that, if the Bernoulli component is entirely driven by tipping, then in both countries tipping can account for more than 50% of total variation of voter turnout. These point estimates suggest that the potential effect sizes of tipping are substantial in both US and UK, but are stronger in UK.

In Figure 6.1 we plot the estimated probability densities for voter turnout in both US and UK, using the estimated parameters from Table 1. It shows that in UK the probability of reaching the higher peak is larger than that in the US. One possible explanation for this in light of our model is that the tipping threshold \bar{V} is lower in UK than in US. Recall that $\bar{V} = (1 - \beta) / (2\kappa)$, a lower \bar{V} may therefore be driven by higher β (i.e., stronger expressive payoff or civic duty of voting) and higher κ (i.e., stronger expressive externality).

Figure 6.1: Estimated densities of turnout distribution for US and UK



Sampling error. It is important to understand whether these economically significant point estimates are simply due to sampling error in what is a relatively small sample. In particular, how likely is it that such large estimates could be generated from an underlying model with no tipping, that is $g = 0$ or $Q \in \{0, 1\}$? We cannot simply apply asymptotic theory here because the hypothesis of no tipping is on the boundary of the parameter space and the distribution of the coefficient estimate \hat{g} is a positive random variable, hence biased away from 0 even when the true value is 0, and does not converge in a large sample to an approximate normal. To understand better the role of sampling error we use a Monte

Carlo experiment; the detailed procedure is explained in Appendix D. Our procedure here is conceptually the same as a randomization or permutation test (Young, 2019) in the sense that we ask how likely it is under the null hypothesis (i.e., $g = 0$ or $Q_1 \in \{0, 1\}$) that we would see coefficient estimates as high as those we estimated (i.e., \hat{g} and $\hat{\sigma}_B$ from Table 1).

Our results (cf. Table 3 in Appendix D) show that for UK data the likelihood of observing estimates equal to or larger than \hat{g} and $\hat{\sigma}_B$ in Table 1 from a model without tipping are both less than 10%. The case for the US data is in sharp contrast: there, the likelihood of obtaining an estimated gap \hat{g} as we found in data or higher is nearly 50%, and for $\hat{\sigma}_B$ this likelihood is more than 22%. These results suggest that the tipping we found in the US is quite likely to be a statistical fluke, while this is relative unlikely for UK. This again confirms our result that tipping is more relevant in UK than in US.

7. Conclusion

We have developed a model of group participation that combines peer pressure with expressive externality and shown how this can lead to “tipping.” This occurs when the expressive externality is strong enough that in the group the marginal cost of mobilizing participation becomes negative so that it is optimal for everyone to participate. We argued that this potentially explains why sporting teams ration tickets: they do not wish to take the chance of triggering “tipping in reverse” by setting the price so high that it does not pay for the fans to self-organize. Notice that in the real world sports teams are quite aware of fan self-organization and work hard to encourage it. Our second application was to voting where we showed how voter turnout can change discontinuously even though the underlying stakes are drawn from a continuous distribution. We estimated the effects of tipping in US and UK through the lens of our model. Our results suggests that tipping is more relevant in UK than in US.

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Appendix A: All Proofs for Section 4

In this appendix we derive the group’s optimal choice of social norm φ_k . We shall assume throughout that $p_k(\phi_k)$ is a linear function of ϕ_k on $[0, 1]$ with slope $\psi \geq 0$. Observe that $\phi_k = \varphi_k$ must hold to be in compliance with any social norm $\varphi_k \geq \underline{\varphi}$. Using (2.1) we get group utility per capita:

$$\mathcal{U}_k(\varphi_k) \equiv p_k(\varphi_k)v_k + \lambda\varphi_k - C(\varphi_k) \quad (7.1)$$

for all $\varphi_k \in [\underline{\varphi}, 1]$. In what follows we prove Lemma 2 and Propositions 1 and 2 in order.

A.1. Proof of Lemma 2

We prove Lemma 2 by establishing a more general result: suppose $\pi = 1/2$ and let

$$\xi \equiv \frac{1 - \beta}{2} - \lambda - \psi v_k. \quad (7.2)$$

Then the per capita group utility $\mathcal{U}_k(\varphi_k)$ is linear in φ_k on $(\underline{\varphi}, 1]$ and $\mathcal{U}'_k(\varphi_k) = -\xi$ for all $\varphi_k > \underline{\varphi}$. Moreover, $\mathcal{U}_k(\cdot)$ is given by

$$\mathcal{U}_k(\varphi_k) = p_k(\varphi_k)v_k + \left[\lambda - \frac{1 - \beta}{2} \right] \varphi_k + \frac{\beta}{2} \cdot \mathbf{1}\{\varphi_k > 0\} \quad \text{for } \varphi_k \in [\underline{\varphi}, 1]. \quad (7.3)$$

Lemma 2 is then a special case of this result when $v_k = 0$. By (3.4) and (7.1), for all $\varphi_k > \underline{\varphi}$ we have

$$\begin{aligned}\mathcal{U}_k(\varphi_k) &= p_k(\varphi_k)v_k + \lambda\varphi_k - C(\varphi_k) \\ &= p_k(\varphi_k)v_k + \lambda\varphi_k - \left(\frac{1-\beta}{2}\varphi_k - \frac{\beta}{2}\right) \\ &= p_k(\varphi_k)v_k + \left[\lambda - \frac{1-\beta}{2}\right]\varphi_k + \frac{\beta}{2}\end{aligned}$$

For $\varphi_k = \underline{\varphi}$, we shall establish that

$$\mathcal{U}_k(\underline{\varphi}) = \begin{cases} p_k(0)v_k & \text{if } \beta < 0 \\ p_k(\underline{\varphi})v_k + \left[\lambda - \frac{1-\beta}{2}\right]\underline{\varphi} + \frac{\beta}{2} & \text{if } \beta \geq 0 \end{cases} \quad (7.4)$$

These two expressions together imply (7.3). By (7.1), $\mathcal{U}_k(\underline{\varphi}) = p_k(\underline{\varphi})v_k + \lambda\underline{\varphi} - C(\underline{\varphi})$. If $\beta < 0$, then $\underline{\varphi} = 0$ and $C(\underline{\varphi}) = 0$ so that $\mathcal{U}_k(\underline{\varphi}) = p_k(0)v_k$. If $\beta \in [0, 1]$, then $\underline{\varphi} = \beta \geq 0$ and $C(\underline{\varphi}) = \beta(\beta - 2\beta)/2 = -\beta^2/2 = -\underline{\varphi}\beta/2$. Therefore, by (7.1),

$$\begin{aligned}\mathcal{U}_k(\underline{\varphi}) &= p_k(\underline{\varphi})v_k + \lambda\underline{\varphi} - \frac{\beta}{2}\underline{\varphi} \\ &= p_k(\underline{\varphi})v_k + \left[\lambda - \frac{1-\beta}{2}\right]\underline{\varphi} + \frac{\underline{\varphi}}{2},\end{aligned}$$

which coincides with (7.4). Finally, if $\beta > 1$ then $\underline{\varphi} = 1$ and $C(\underline{\varphi}) = 1/2 - \beta$ so that $\mathcal{U}_k(\underline{\varphi}) = p_k(1)v_k + \lambda - 1/2 + \beta$. It again coincides with (7.4) for $\underline{\varphi} = 1$. These together establish (7.4), which implies $\mathcal{U}'_k(\varphi_k) = -\xi$ for all $\varphi_k > \underline{\varphi}$ straightforwardly.

A.2. Proof of Proposition 1

We maintain the simplification assumptions $\pi = 1/2$ and $v_k = 0$. Recall that

$$\lambda^* \equiv \begin{cases} \frac{1-\beta}{2} & \text{if } \beta \geq 0 \\ \frac{1}{2} - \beta & \text{if } \beta < 0 \end{cases}$$

and $\varphi_k^* = \arg \max_{\varphi_k \in [\underline{\varphi}, 1]} \{\mathcal{U}_k(\varphi_k)\}$. We now prove Proposition 1, that tipping takes place at λ^* .

First observe that if $\beta > 1$ then $\underline{\varphi} = 1$ (cf. (2.2)) so that $\varphi_k^* = 1$ holds trivially. Assume then $\beta \leq 1$. It follows from Lemma 2 that $\mathcal{U}_k(\varphi_k)$ is linear in φ_k for $\varphi_k > \underline{\varphi}$ so that φ_k^* must be either $\underline{\varphi}$ or 1 whenever $\xi \neq 0$. Therefore, $\varphi_k^* = 1$ if $\mathcal{U}_k(1) > \mathcal{U}_k(\underline{\varphi})$ and $\varphi_k^* = \underline{\varphi}$ if

$\mathcal{U}_k(1) < \mathcal{U}_k(\underline{\varphi})$. By (4.2) and (4.1), we have

$$\mathcal{U}_k(1) - \mathcal{U}_k(\underline{\varphi}) = \begin{cases} -\xi(1 - \underline{\varphi}) & \text{if } \beta \geq 0 \\ -\xi + \beta/2 & \text{if } \beta < 0 \end{cases}$$

Therefore when $\beta \geq 0$ we have $\varphi_k^* = \underline{\varphi}$ if $\xi > 0$ and $\varphi_k^* = 1$ if $\xi < 0$. By (4.1) $\xi = (1 - \beta)/2 - \lambda$ is strictly decreasing in λ and it is zero if and only if $\lambda = (1 - \beta)/2$. This proves the result for the case $\beta \geq 0$.

Now consider $\beta < 0$ (where \mathcal{U}_k has a downward jump at zero). In this case we have $\mathcal{U}_k(1) - \mathcal{U}_k(0) = -\xi + \beta/2 = \lambda + \beta - 1/2$, which is increasing in λ and it equals 0 for $\lambda = 1/2 - \beta$. This proves (4.4) for the case $\beta < 0$. That λ^* is decreasing in b follows immediately from its definition.

A.3. Proof of Proposition 2

For this proposition we allow for $\pi \neq 1/2$ and $v_k > 0$, and we assume that $p'_k(\cdot) = \psi \geq 0$. Let $\varphi_k^* = \arg \max_{\varphi_k \in [\underline{\varphi}, 1]} \{\mathcal{U}_k(\varphi_k)\}$ and observe that $\varphi_k^* = 1$ must hold when $\beta \geq 1$ because $\underline{\varphi} = 1$ (cf. (2.2)), so we focus on $\beta < 1$ (whence $\underline{\varphi} < 1$) from now on. Using the formulas of $C(\varphi_k)$ and $\mathcal{U}_k(\varphi_k)$ (cf. (3.3) and (7.1)), for $\varphi_k > \underline{\varphi}$ we have

$$C'(\varphi_k) = \pi(1 - \varphi_k) + (1 - \pi)(\varphi_k - \beta) \quad (7.5)$$

$$\begin{aligned} \mathcal{U}'_k(\varphi_k) &= \psi v_k + \lambda - C'(\varphi_k) \\ &= \psi v_k + \lambda - \pi(1 - \varphi_k) - (1 - \pi)(\varphi_k - \beta) \end{aligned} \quad (7.6)$$

Suppose φ_k^* is interior so that $\varphi_k^* \in (\underline{\varphi}, 1)$; then $\mathcal{U}'_k(\varphi_k^*) = 0$ and $\mathcal{U}''_k(\varphi_k^*) \leq 0$ must hold. For $\pi > 1/2$, however, $\mathcal{U}''_k(\varphi_k) = 2\pi - 1 > 0$ and thus $\varphi_k^* \in (\underline{\varphi}, 1)$ cannot hold. Therefore, φ_k^* must be a corner solution and it equals 1 or $\underline{\varphi}$ if $\mathcal{U}_k(1)$ is respectively larger or smaller than $\mathcal{U}_k(\underline{\varphi})$. From equations (3.3) and (7.1) simple algebra shows that $\mathcal{U}_k(1) - \mathcal{U}_k(\underline{\varphi}) > 0$ if and only if $\lambda > \lambda^{**} = \lambda^* - \psi v_k$. So for $\pi > 1/2$ and $\psi \geq 0$ the tipping result is the same as in the case of $\pi = 1/2$ except that the tipping point λ^* is shifted down by ψv_k . This proves part (1) of Proposition 2.

To establish part (2), now consider $\pi < 1/2$. We start by assuming $\beta \geq 0$ to prove the statement in the text. In this case \mathcal{U}_k is continuous on $[\underline{\varphi}, 1]$ and strictly concave so the optimum φ_k^* is equal to $\underline{\varphi} = \beta$ if $\mathcal{U}'_k(\underline{\varphi}) \leq 0$, interior if $\mathcal{U}'_k(\underline{\varphi}) > 0 > \mathcal{U}'_k(1)$, and equal to 1 if $\mathcal{U}'_k(1) \geq 0$. It is easy to verify that

$$\begin{aligned} \mathcal{U}'_k(\underline{\varphi}) \leq 0 &\iff \lambda \leq 2\pi\lambda^* - \psi v_k \\ \mathcal{U}'_k(1) \geq 0 &\iff \lambda \geq 2(1 - \pi)\lambda^* - \psi v_k. \end{aligned}$$

Recall that $\lambda^{**} = \lambda^* - \psi v_k$. Then for all $\pi < 1/2$ we have $2\pi\lambda^* - \psi v_k/h < \lambda^{**} < 2(1 - \pi)\lambda^* + \psi v_k$ and both boundaries converge to λ^{**} as $\pi \nearrow 1/2$. The stationary point $\mathcal{U}'_k(\varphi_k^o) = 0$ being

$$\varphi_k^o(\lambda) = \frac{\lambda + \psi v_k + (1 - \pi)\beta - \pi}{1 - 2\pi} = \frac{\lambda - \lambda^{**}}{1 - 2\pi} + \frac{1 + \varphi}{2}, \quad (7.7)$$

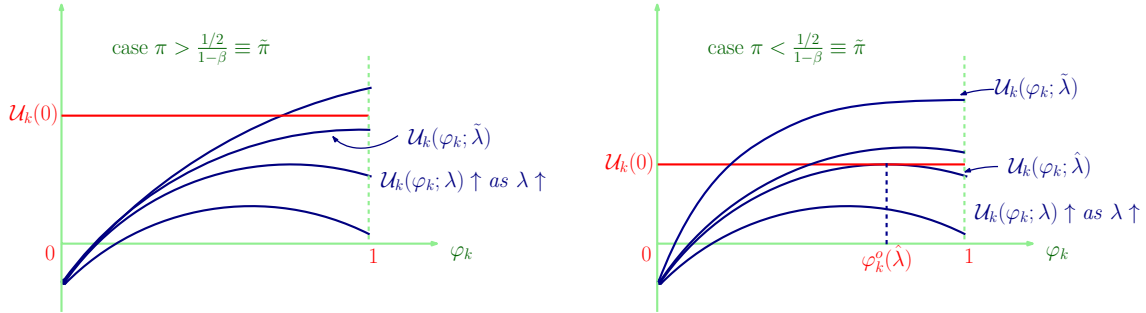
the optimal solution is as follows:

$$\varphi_k^* = \begin{cases} \varphi & \text{if } \lambda \leq 2\pi\lambda^* - \psi v_k \\ \frac{\lambda - \lambda^{**}}{1 - 2\pi} + \frac{1 + \varphi}{2} & \text{if } \lambda \in (2\pi\lambda^* - \psi v_k, 2(1 - \pi)\lambda^* - \psi v_k) \\ 1 & \text{if } \lambda \geq 2(1 - \pi)\lambda^* - \psi v_k \end{cases} \quad (7.8)$$

Plugging $\pi = 1/2 - \epsilon$ for $\epsilon \in (0, 1/2)$ in to (7.8) yields (4.5). This ends the proof of the statements in Proposition 2 in Section 4.

We turn now to the case $\beta < 0$ and $\pi < 1/2$ for completeness. In this case we have $\varphi = 0$, $\mathcal{U}_k(0) = p_k(0)v_k$ and $\mathcal{U}_k(\varphi_k)$ has a downward jump at zero. Before formally stating the results, it is helpful to visualize the situation first:

Figure 7.1: $\mathcal{U}_k(\varphi_k; \lambda)$



In what follows, we write $\mathcal{U}_k(\varphi_k)$ as $\mathcal{U}_k(\varphi_k; \lambda)$ for all $\varphi_k > 0$ to make explicit its dependence on λ . Figure 7.1 depicts the family of functions $\mathcal{U}_k(\varphi_k; \lambda)$ depending on whether π lies above or below a threshold

$$\tilde{\pi} \equiv \frac{1}{2} \frac{1}{1 - \beta}.$$

Two observations are critical to the analysis. First, $\mathcal{U}_k(\varphi_k; \lambda)$ strictly increases in λ for all $\varphi_k > 0$ because $\partial \mathcal{U}_k(\varphi_k; \lambda) / \partial \lambda = \varphi_k > 0$. Second, as is clear from (7.7), the stationary point $\varphi_k^o(\lambda)$ is continuously increasing in λ and there exists a unique threshold

$$\tilde{\lambda} \equiv (1 - \pi)(1 - \beta) - \psi v_k$$

such that $\varphi_k^o(\tilde{\lambda}) = 1$ (or equivalently $\mathcal{U}'_k(1; \tilde{\lambda}) = 0$). As the left panel of Figure 7.1 shows, for $\pi > \tilde{\pi}$ it holds that $\mathcal{U}_k(1; \tilde{\lambda}) < \mathcal{U}_k(0)$ so that $\varphi_k^* = 0$ for λ close to $\hat{\lambda}$; as λ grows further $\mathcal{U}_k(1; \lambda)$ crosses $\mathcal{U}_k(0)$ at some threshold and φ_k^* jumps to 1. We will show that this threshold coincides with $\lambda^{**} = \frac{1}{2} - \beta - \psi v_k$ so that the tipping result in Proposition 2 still applies. The right panel of Figure 7.1 suggests that for $\pi < \tilde{\pi}$ we have $\mathcal{U}_k(1; \tilde{\lambda}) > \mathcal{U}_k(0)$ so that for λ slightly below $\tilde{\lambda}$ the optimal solution φ_k^* is given by $\varphi_k^o(\lambda) \in (0, 1)$ and the value equals $\mathcal{U}_k(\varphi_k^o(\lambda); \lambda)$; as λ decreases further $\mathcal{U}_k(\varphi_k^o(\lambda); \lambda)$ strictly decreases and there exists a threshold $\hat{\lambda}$ (derived below) such that $\mathcal{U}_k(\varphi_k^o(\hat{\lambda}); \hat{\lambda}) = 0$. Hence, φ_k^* drops discontinuously from $\varphi_k^o(\hat{\lambda})$ to 0 as λ crosses $\hat{\lambda}$ from above; whence the tipping result holds.

In the remainder of this appendix we formally derive φ_k^* for the two cases: $\pi \geq \tilde{\pi}$ and $\pi < \tilde{\pi}$. To begin with, it is useful to observe that $\tilde{\lambda} < \lambda^{**}$ if and only if $\pi > \tilde{\pi}$ and that $\mathcal{U}_k(1; \lambda) < \mathcal{U}_k(0)$ if and only if $\lambda < \lambda^{**}$.

Case 1: $\pi > \tilde{\pi}$ so that $\tilde{\lambda} < \lambda^{**}$. For $\lambda > \lambda^{**}$, it follows from previous arguments that $\mathcal{U}_k(\varphi_k; \lambda)$ is strictly increasing in φ_k on $(0, 1]$ and $\mathcal{U}_k(1; \lambda) > \mathcal{U}_k(0)$. Therefore, $\varphi_k^* = 1$. For $\lambda < \lambda^{**}$, we argue that $\mathcal{U}_k(0) > \mathcal{U}_k(\varphi_k; \lambda)$ for all $\varphi_k \in (0, 1]$ and therefore $\varphi_k^* = 0$. To see this, notice that $\mathcal{U}_k(\varphi_k; \lambda)$ is strictly increasing in λ for all $\varphi_k > 0$ and it is strictly increasing in φ_k for $\lambda = \lambda^{**} > \tilde{\lambda}$. Hence, for all $\varphi_k \in (0, 1]$ and $\lambda < \lambda^{**}$ we have

$$\mathcal{U}_k(\varphi_k; \lambda) < \mathcal{U}_k(\varphi_k; \lambda^{**}) \leq \mathcal{U}_k(1; \lambda^{**}) = \mathcal{U}_k(0).$$

Taken together, when $\pi > \tilde{\pi}$ it holds that $\varphi_k^* = 1$ for $\lambda > \lambda^{**}$ and $\varphi_k^* = 0$ for $\lambda < \lambda^{**}$, which coincides with part (1) of Proposition 2.

Case 2: $\pi < \tilde{\pi}$ so that $\tilde{\lambda} > \lambda^{**}$. For this case, we shall establish that

$$\varphi_k^* = \begin{cases} 0 & \text{if } \lambda < \hat{\lambda} \\ \frac{\psi v_k + \lambda + (1 - \pi)\beta - \pi}{1 - 2\pi} & \text{if } \lambda \in [\hat{\lambda}, \tilde{\lambda}] \\ 1 & \text{if } \lambda > \tilde{\lambda} \end{cases} \quad (7.9)$$

where

$$\hat{\lambda} \equiv \pi - (1 - \pi)\beta - \psi v_k + 2\sqrt{-\pi(1/2 - \pi)\beta}.$$

It can be verified $\hat{\lambda} < \lambda^{**} < \tilde{\lambda}$ when $\pi < \tilde{\pi}$. Moreover, for $\lambda = \hat{\lambda}$, we have

$$\varphi_k^* = \frac{\psi v_k + \hat{\lambda} + (1 - \pi)\beta - \pi}{1 - 2\pi} = \sqrt{\frac{\pi\beta}{\pi - 1/2}} \in (0, 1)$$

for $\pi \in (0, \tilde{\pi})$. Therefore, in the same spirit of Proposition 1 for the case of $\pi = 1/2$, φ_k^* increases discontinuously as λ crosses a threshold $\hat{\lambda}$. To show (7.9), first consider $\lambda \geq \tilde{\lambda}$. In this case, $\mathcal{U}_k(\varphi_k; \lambda)$ is strictly increasing on $(0, 1]$ and $\mathcal{U}_k(1; \lambda) > \mathcal{U}_k(0)$ because $\lambda \geq \tilde{\lambda} > \lambda^{**}$.

Hence $\varphi_k^* = 1$. Below we assume $\lambda < \tilde{\lambda}$ such that φ_k^* can only be $\varphi_k^o(\lambda)$ (if positive) or 0. Notice that $\varphi_k^o(\lambda) \in (0, 1)$ if and only if $\mathcal{U}'_k(1; \lambda) < 0 < \lim_{\varphi_k \downarrow 0} \mathcal{U}'_k(\varphi_k; \lambda)$. $\mathcal{U}'_k(1; \lambda) < 0$ is equivalent to $\lambda < \tilde{\lambda}$. $0 < \lim_{\varphi_k \downarrow 0} \mathcal{U}'_k(\varphi_k; \lambda)$ holds if $\psi v_k + \lambda - \pi + (1 - \pi)\beta > 0$, or equivalently,

$$\lambda > \pi - (1 - \pi)\beta - \psi v_k \equiv \underline{\lambda}.$$

Hence, $\varphi_k^o(\lambda) \in (0, 1)$ if and only if $\underline{\lambda} < \lambda < \tilde{\lambda}$, and it holds that $\varphi_k^o(\lambda) = 1$ for $\lambda = \tilde{\lambda}$ and $\varphi_k^o(\lambda) \downarrow 0$ for $\lambda \downarrow \underline{\lambda}$. Assume $\underline{\lambda} < \lambda < \tilde{\lambda}$. For $\varphi_k^o(\lambda)$ to be globally optimal, it must hold that $\mathcal{U}_k(\varphi_k^o(\lambda); \lambda) \geq \mathcal{U}_k(0)$. Because $\mathcal{U}_k(\varphi_k^o(\lambda); \lambda)$ is continuous and strictly increasing in λ and it satisfies $\mathcal{U}_k(\varphi_k^o(\tilde{\lambda}); \tilde{\lambda}) = \mathcal{U}_k(1; \tilde{\lambda}) > \mathcal{U}_k(0)$ and $\lim_{\lambda \downarrow \underline{\lambda}} \mathcal{U}_k(\varphi_k^o(\lambda); \lambda) = (h - c)/2 < 0$, there exists a unique threshold $\lambda \in (\underline{\lambda}, \tilde{\lambda})$ such that $\mathcal{U}_k(\varphi_k^o(\lambda); \lambda) = 0$. It can be verified that this threshold is precisely $\hat{\lambda}$ defined above and it indeed satisfies $\underline{\lambda} < \hat{\lambda} < \tilde{\lambda}$. Therefore, $\mathcal{U}_k(\varphi_k^o(\lambda); \lambda) < 0$ and thus $\varphi_k^* = 0$ for $\lambda \in (\underline{\lambda}, \hat{\lambda})$, while $\mathcal{U}_k(\varphi_k^o(\lambda); \lambda) > 0$ and $\varphi_k^* = \varphi_k^o(\lambda)$ for $\lambda \in (\hat{\lambda}, \tilde{\lambda})$. Finally, for $\lambda \leq \underline{\lambda}$, it holds that $\mathcal{U}_k(\varphi_k; \lambda)$ is strictly decreasing in φ_k and hence $\varphi_k^* = 0$. Combining these together, we obtain (7.9), which suggests that φ_k^* increases linearly from $\varphi = 0$ to 1 as λ crosses the interval $(\underline{\lambda}, \tilde{\lambda})$. Let $\pi = 1/2 - \epsilon$, then as $\epsilon \rightarrow 0$ we have $\tilde{\lambda} - \underline{\lambda} \rightarrow 0$ so that the width of this interval vanishes to zero. This result is reminiscent of part (2) of Proposition (2).

Appendix B: Proof of Proposition 3

Recall from (5.2) that when tipping is impossible the standard monopoly optimal pricing is to charge $r^M(Q) = \beta_0 - Q$ for $Q < \beta_0/2$ and $r^M(Q) = \beta_0/2$ for $Q \geq \beta_0/2$. The resulting profit is

$$\Pi^M(Q) = \begin{cases} (\beta_0 - Q)Q & \text{if } Q < \beta_0/2 \\ \beta_0^2/4 & \text{if } Q \geq \beta_0/2 \end{cases}. \quad (7.10)$$

Now suppose that tipping is possible. Let

$$\hat{\lambda}_Q \equiv \begin{cases} \frac{1-Q}{2} - \psi v_k & \text{if } Q < \beta_0/2 \\ \frac{1}{2} \left[\left(1 - \frac{\beta_0}{2}\right)^2 + \left(\frac{1}{Q} - 1\right) \left(\frac{\beta_0}{2}\right)^2 \right] - \psi v_k & \text{if } Q \geq \beta_0/2 \end{cases} \quad (7.11)$$

It is routine to verify that $\hat{\lambda}_Q$ is strictly decreasing in Q with

$$\hat{\lambda}_0 = \frac{1}{2} - \psi v_k = \bar{\lambda} \quad \text{and} \quad \hat{\lambda}_1 = \frac{1}{2} \left(1 - \frac{\beta_0}{2}\right)^2 - \psi v_k = \underline{\lambda}. \quad (7.12)$$

We show that

Lemma 3. Suppose $\pi = 1/2$ and $p'_k(\cdot) = \psi \geq 0$. For any fixed $Q \in [0, 1]$, the following properties hold:

1. If $\lambda < \hat{\lambda}_Q$ then charging price $r^M(Q)$ without triggering tipping is optimal.
2. If $\lambda \geq \hat{\lambda}_Q$ then it is optimal for the sports team to trigger tipping and sell at full capacity Q by charging price

$$r_Q^* = \begin{cases} \beta_0 + 2(\lambda + \psi v_k) - 1 & \text{if } \hat{\lambda}_Q \leq \lambda < \bar{\lambda} \\ \beta_0 + (2(\lambda + \psi v_k) - 1) \frac{Q}{1+Q} & \text{if } \lambda \geq \bar{\lambda} \end{cases}. \quad (7.13)$$

Essentially, Lemma 3 says that for any fixed Q there is a critical cutoff $\hat{\lambda}_Q$ for expressive externality λ such that the optimal ticket price triggers tipping if and only if $\lambda \geq \hat{\lambda}_Q$. Moreover, the tipping price r_Q^* is non-decreasing in capacity Q : it is invariant in Q for $\lambda \in [\hat{\lambda}_Q, \bar{\lambda}]$ and is strictly increasing in Q for $\lambda > \bar{\lambda}$.

Proof of Lemma 3. The key observation is that to trigger tipping so that all fans participate the following two conditions must be satisfied:

- (i) *Incentive compatibility:* the marginal cost must be non-positive, that is $\xi \leq 0$.
- (ii) *Group rationality:* the group payoff from participation at maximum capacity Q must be higher than without self-organization, that is $\mathcal{U}_k(Q) - \mathcal{U}_k(\varphi) \geq 0$.

By (7.2) and $\beta = \beta_0 - r$ we have

$$\xi = \frac{1}{2} - \frac{\beta_0 - r}{2} - \lambda - \psi v_k. \quad (7.14)$$

Therefore $\xi \leq 0$ if and only if

$$r \leq r^I \equiv \beta_0 + 2(\lambda + \psi v_k) - 1$$

There would be no point in using prices higher than this: only the committed fans would buy the ticket, and $r > r^I$ is equivalent to $\beta_0 - r < 1 - 2(\lambda + \psi v_k)$ so if $2\lambda \geq 1 - 2\psi v_k$ this means selling no tickets at all (not optimal), while if $2\lambda < 1 - 2\psi v_k$ then profit would be $r(\beta_0 - r)$ which is the same as the monopoly problem.

At price r^I we have $b = \beta_0 - r^I = 1 - 2(\lambda + \psi v_k) \geq 0$ if and only if $\lambda + \psi v_k \leq 1/2$. Therefore, by (5.1), the fraction of committed voters under price r^I is given by

$$\underline{\varphi} = \begin{cases} 1 - 2(\lambda + \psi v_k) & \text{if } \lambda + \psi v_k \leq 1/2 \\ 0 & \text{if } \lambda + \psi v_k > 1/2 \end{cases}$$

Note that if $1 - 2(\lambda + \psi v_k) \geq Q$ (or equivalently $2\lambda \leq 1 - Q - 2\psi v_k$), then $\underline{\varphi} \geq Q$ and

$r^I \leq \beta - Q$ so that the profit must be bounded above by $r^I Q \leq (\beta_0 - Q) Q \leq \Pi^M(Q)$. Therefore, for $2\lambda \leq 1 - Q - 2\psi v_k$ the monopoly solution is optimal. In what follows we then assume $2\lambda > 1 - Q - 2\psi v_k$ so that $\underline{\varphi} < Q$. From here on we distinguish two cases: $\lambda \leq 1/2 - \psi v_k$ and $\lambda > 1/2 - \psi v_k$, or equivalently $\lambda \leq \bar{\lambda}$ and $\lambda > \bar{\lambda}$.

Case 1: $\lambda \leq \bar{\lambda}$ In this case $\mathcal{U}_k(\varphi_k)$ is linear in φ_k on $(\underline{\varphi}, 1]$ and is continuous at $\underline{\varphi}$. By Proposition 1 participation at maximal capacity Q occurs only if $\xi \leq 0$, or equivalently $r \leq r^I$. r^I is thus the highest price the seller can charge that induce participation rate Q under self-organization, in which case the profit is $r^I Q$. It remains to compare this profit with the monopoly profit $\Pi^M(Q)$. If $Q \leq \beta_0/2$, we have $\Pi^M(Q) = (\beta_0 - Q) Q < r^I Q$ because $r^I = \beta_0 + 2(\lambda + \psi v_k) - 1 > \beta_0 - Q$ for all $\lambda > (1 - Q)/2 - \psi v_k$. The optimal solution is then to sell at capacity Q with price r^I for all $\lambda > (1 - Q)/2 - \psi v_k$. If $Q > \beta_0/2$, then $\Pi^M(Q) = \beta_0^2/4$. Simple algebra shows that $r^I Q \geq \beta_0^2/4$ if and only if

$$\lambda > \frac{1}{2} \cdot \left[\left(\frac{\beta_0}{2} - 1 \right)^2 + \left(\frac{1}{Q} - 1 \right) \left(\frac{\beta_0}{2} \right)^2 \right] - \psi v_k = \hat{\lambda}_Q$$

Moreover, for all $\beta_0 \in [0, 2Q]$ it can be verified that

$$1 - Q < \left(\frac{\beta_0}{2} - 1 \right)^2 + \left(\frac{1}{Q} - 1 \right) \left(\frac{\beta_0}{2} \right)^2 \leq 1$$

Hence, when $\beta_0/2 \leq Q$, it is optimal to sell at full capacity Q if $\lambda \in [\hat{\lambda}_Q, \bar{\lambda}]$. Below we will show that the same is also true for all $\lambda > \bar{\lambda}$. The conclusion will be that when $\beta_0/2 \leq Q$ it is optimal to sell at full capacity for all $\lambda \geq \hat{\lambda}_Q$.

Case 2: $\lambda > \bar{\lambda}$ In this case we have $\beta = \beta_0 - r^I < 0$. Hence, $\underline{\varphi} = 0$ and there is a discontinuous drop of $\mathcal{U}_k(\varphi_k)$ at 0. For selling at maximum capacity Q to be optimal, the price r must satisfy the group rationality constraint that $U_k(0) \leq \mathcal{U}_k(Q)$. Simple algebra shows that this condition is equivalent to

$$r \leq \beta_0 + [2(\psi v_k + \lambda) - 1] \frac{Q}{1 + Q} \equiv r^{II}$$

Notice that r^{II} is increasing in Q so that the capacity limit Q does restrict the price the firm can charge. Again, for r^{II} to be optimal, it must hold that $r^{II} Q > \Pi^M(Q)$. If $\beta_0/2 > Q$, then $r^{II} Q > \Pi^M(Q)$ if and only if $r^{II} > \beta_0 - Q$, which always holds because $2(\lambda + \psi v_k) - 1 > 0$ for all $\lambda > \bar{\lambda}$ by definition of $\bar{\lambda}$. If $\beta_0/2 \leq Q$, then $\Pi^M(Q) = \beta_0^2/4$ and

$r^{II}Q > \Pi^M(Q)$ holds if and only if

$$\lambda > \frac{1}{2} \left\{ (1+Q) \left[\left(\frac{\beta_0}{2Q} - 1 \right)^2 - 1 \right] + 1 - 2\psi v_k \right\}$$

Notice that the right-hand side of the above inequality is strictly smaller than $\bar{\lambda}$ for $\beta_0/2 < Q$, because $\left(1 - \frac{\beta_0}{2Q}\right)^2 - 1 < 0$. Therefore, selling at the maximum capacity at price r^{II} , which is strictly increasing in Q , is optimal for all $\lambda > \bar{\lambda}$.

Combining these together: it is optimal to exploit the self-organization of fans and sell at capacity Q if and only if $\lambda \geq \hat{\lambda}_Q$ and the optimal tipping price r_Q^* equals r^I for $\lambda < \bar{\lambda}$ and r^{II} for $\lambda \geq \bar{\lambda}$. \square

With the help of Lemma 3 we are now ready to prove Proposition 3.

Proof of Proposition 3. First of all, recall from (7.11) and (7.12) that the $\hat{\lambda}_Q$ is a continuous and strictly decreasing function of Q and satisfy $\hat{\lambda}_0 = \bar{\lambda}$ and $\hat{\lambda}_1 = \underline{\lambda}$. Then, by Lemma 3, if $\lambda \leq \underline{\lambda}$ the sports team's optimal strategy is to charge the standard monopoly price $r^M(Q)$ without triggering tipping for all $Q \in [0, 1]$; this proves part (1). If instead $\lambda \geq \bar{\lambda}$ then it is always optimal to induce tipping independent of Q and by (7.13) the optimal price $r^*(Q; \lambda) = \beta_0 + [2(\psi v_k + \lambda) - 1] \frac{Q}{1+Q}$ is strictly increasing in Q ; this proves part (2).

Now we consider intermediate $\lambda \in (\underline{\lambda}, \bar{\lambda})$ and establish part (2). By Lemma 3, for all $\lambda < \bar{\lambda}$ the tipping price is $\beta_0 + 2(\lambda + \psi v_k) - 1$ so that the profit when tipping is triggered is $\Pi^T(Q; \lambda) = (\beta_0 + 2(\lambda + \psi v_k) - 1)Q$. Recall that the optimal profit $\Pi^M(Q)$ when tipping is impossible is given by (7.10). Simple algebra shows that $[\Pi^T(Q; \lambda) - \Pi^M(Q)]/Q$ is continuous and strictly increasing in Q and λ , and it is strictly negative for $Q \downarrow 0$ and positive for $Q \uparrow 1$ when $\lambda \in (\underline{\lambda}, \bar{\lambda})$. There thus exists a unique $Q_\lambda^* \in (0, 1)$ such that $\Pi^T(Q_\lambda^*; \lambda) = \Pi^M(Q_\lambda^*)$ and this Q_λ^* is continuous and strictly decreasing in λ . Solving this profit indifference condition yields

$$Q_\lambda^* = \begin{cases} \left(\frac{\beta_0}{2} \right)^2 / (\beta_0 + 2(\lambda + \psi v_k) - 1) & \text{if } \lambda \in (\underline{\lambda}, \tilde{\lambda}) \\ 1 - 2(\lambda + \psi v_k) & \text{if } \lambda \in [\tilde{\lambda}, \bar{\lambda}) \end{cases}$$

where $\tilde{\lambda} = \frac{1}{2} \left(1 - \frac{\beta_0}{2}\right) - \psi v_k$. It is easy to verify that (i) $Q \geq Q_\lambda^*$ if and only if $\lambda \geq \hat{\lambda}_Q$, and (ii) Q_λ^* is continuous and strictly decreasing in λ on $(\underline{\lambda}, \bar{\lambda})$ and $Q_{\bar{\lambda}}^* = \beta_0/2$. Therefore, by (i) and Lemma 3, triggering tipping is optimal if and only if $Q \geq \hat{Q}_\lambda$. The optimal price is then

$$r^*(Q; \lambda) = \begin{cases} r^M(Q) & \text{if } Q < Q_\lambda^* \\ \beta_0 + 2(\lambda + \psi v_k) - 1 & \text{if } Q \geq Q_\lambda^* \end{cases}.$$

Finally, recall that $r^M(Q) = \max\{\beta_0 - Q, \beta_0/2\}$. Combine this together with the expression above and the definitions of $\tilde{\lambda}$ and Q_λ^* , we obtain that $r^*(Q; \lambda)$ is weakly decreasing in Q and there is a discontinuous downward jump at Q_λ^* if and only if $\lambda < \tilde{\lambda}$. This completes the proof of part (2). \square

Appendix C: Proof of Proposition 4

The following lemma characterizes the per capita utility $\mathcal{U}_k(b_k, b_{-k}, V)$ for party $k = L, S$ as a function of both parties' bids and the total prize V .

Lemma 4. *Suppose $\pi = 1/2$, $\beta \in (0, 1)$ and $\lambda = \kappa V$ with $\kappa > 0$. Then*

$$\mathcal{U}_k(b_k, b_{-k}, V) = \Pi_k(b_k, b_{-k}) \frac{V}{\eta_k} - \xi(V) \frac{b_k}{\eta_k} + \frac{\beta}{2}$$

where

$$\xi(V) \equiv \frac{1 - \beta}{2} - \kappa V$$

is the marginal cost of increasing turnout rate $\varphi_k = b_k/\eta_k$ for party k and it is decreasing in total prize V .

Proof. When $\beta \in (0, 1)$ it follows from (2.2) that $\varphi = \beta \in (0, 1)$. Consider any implementable turnout rate $\varphi_k \in [\beta, 1]$ for party k . By (2.1) and (3.4), the per capita utility of party k is given by

$$\begin{aligned} \mathcal{U}_k(\varphi_k) &= p_k(\varphi_k)v_k + \lambda\varphi_k - C(\varphi_k) \\ &= p_k(\varphi_k)v_k - \left(\frac{1 - \beta}{2} - \lambda\right)\varphi_k + \frac{\beta}{2} \end{aligned}$$

Recall that $v_k = V/\eta_k$, $\varphi_k = b_k/\eta_k$, $\lambda = \kappa V$ and $p_k(\varphi_k) = \Pi_k(\eta_k\varphi_k, \eta_{-k}\varphi_{-k})$ for $k = L, S$. Plugging these into $\mathcal{U}_k(\varphi_k)$ yields the statements in this lemma. \square

Lemma 4 suggests that under our simplification assumptions the marginal cost $\xi(V)$ of mobilizing turnout is decreasing in V and it becomes negative for V larger than \bar{V} .

We first establish that for $V > \bar{V}$ there exists a unique equilibrium in dominant strategy in which both parties bid their maxima (i.e., $b_k = \eta_k$ for $k = L, S$) so that $\bar{b} = b_L + b_S = \eta_L + \eta_S$. By Lemma 4, we have

$$\mathcal{U}_k(b_k, b_{-k}, V) = \Pi_k(b_k, b_{-k}) \frac{V}{\eta_k} - \kappa(\bar{V} - V) \frac{b_k}{\eta_k} + \frac{\beta}{2} \quad (7.15)$$

The second step follows from the fact that $\varphi = b$ and $\xi(V) = \kappa(\bar{V} - V)$. Observe for $V > \bar{V}$ that $\mathcal{U}_k(b_k, b_{-k}, V)$ is strictly increasing in b_k because $\Pi_k(b_k, b_{-k})$ is non-decreasing in b_k

and $\kappa(\bar{V} - V)$ is strictly negative. Hence, it is a strictly dominant strategy for each party to bid is maximum and this yields the unique equilibrium.²⁹ This proves the statement in Proposition 4 for $V > \bar{V}$.

In what follows we assume $\eta_S > \beta\eta_L$ and $V < \bar{V}$ so that the marginal cost $\kappa(\bar{V} - V)$ is strictly positive. We will exploit results from Levine and Mattozzi (2020) to establish that for V smaller than but close to \bar{V} there exists a unique equilibrium in mixed strategy. Moreover, in this equilibrium party L bids almost surely η_S while party S bids almost surely $\beta\eta_S$ so that the total bid is almost surely $(1 + \beta)\eta_S$. This then completes the proof for Proposition 4.

We introduce a few definitions and notations. For each party $k \in \{L, S\}$, we define its *desire to bid* $B_k(V)$ as the highest for which party k prefers to get the prize V for sure to bidding $\eta_k \underline{\varphi} = \eta_k \beta$ and get no prize. By (7.15), $B_k(V)$ is given by the solution b_k to

$$\frac{V}{\eta_k} - \xi(V) \frac{b_k}{\eta_k} + \frac{\beta}{2} = -\xi(V)\beta + \frac{\beta}{2}.$$

This yields

$$B_k(V) = \frac{V}{\xi(V)} + \beta\eta_k.$$

We further define party k 's *willingness-to-bid* as $W_k(V) = \min\{B_k(V), \eta_k\}$; this equals the the maximum bid party k is willing or afford to pay. Since $\underline{\varphi} = \beta > 0$ and $\eta_L > \eta_S$, we have $W_S(V) < W_L(V)$; that is, party S is the *disadvantaged group* who has the lower willingness-to-bid. Finally, we let \underline{V} denote the lowest level of prize V such that the disadvantage party S is just indifferent between winning prize V for sure with bid $\eta_L\beta$ (i.e., the smallest bid of the advantaged party L) and bidding $\eta_S\beta$ and get no prize. Therefore, \underline{V} is the solution V to

$$\frac{V}{\eta_S} - \xi(V) \frac{\eta_L}{\eta_S} \beta + \frac{\beta}{2} = -\xi(V)\beta + \frac{\beta}{2}.$$

Using the fact that $\xi(V) = \kappa(\bar{V} - V)$, we obtain

$$\underline{V} = \bar{V} \frac{\kappa\beta(\eta_L - \eta_S)}{1 + \kappa\beta(\eta_L - \eta_S)} < \bar{V}.$$

We denote each party k 's (mixed) bidding strategy by F_k , a cdf on $[\beta\eta_k, \eta_k]$. The following lemma follows from Levine and Mattozzi (2020).

Lemma 5. (Levine and Mattozzi, 2020) Suppose $\underline{V} < V < \bar{V}$, $\beta \in (0, 1)$ and $\eta_S > \beta\eta_L$.

²⁹If $V = \bar{V}$, then $\xi(V) = 0$ and $U_k(b_k, b_{-k}, V)$ is weakly increasing in b_k . Hence, bidding the maximum is still the unique equilibrium in pure and weakly dominant strategies.

Then there is a unique equilibrium in which both parties play the mixed strategies given by

$$F_L(x) = \begin{cases} 1 & \text{if } x \geq W_S(V) \\ \frac{\xi(V)}{V} (x - \beta\eta_S) & \text{if } x \in [\beta\eta_L, W_S(V)) \\ 0 & \text{if } x < \beta\eta_L \end{cases} \quad (7.16)$$

$$F_S(x) = \begin{cases} 1 & \text{if } x \geq W_S(V) \\ 1 - \frac{\xi(V)}{V} (W_S(V) - x) & \text{if } x \in [\beta\eta_L, W_S(V)) \\ 1 - \frac{\xi(V)}{V} (W_S(V) - \beta\eta_L) & \text{if } x \in [\beta\eta_S, \beta\eta_L) \\ 0 & \text{if } x < \beta\eta_S \end{cases} \quad (7.17)$$

The aggregate bid b is the sum of two independent random variables with $b_k \sim F_k$ for $k \in \{L, S\}$.

Proof. This lemma is a direct application of Theorem 1 in the Online Appendix of Levine and Mattozzi (2020) to our model. \square

Since $\lim_{V \nearrow \bar{V}} \xi(V) = \lim_{V \nearrow \bar{V}} \kappa(\bar{V} - V) = 0$, so $\lim_{V \nearrow \bar{V}} V/\xi(V) = \infty$. Therefore for V sufficiently close to \bar{V} we have $W_S(V) = \min\{V/\xi(V) + \eta_S\beta, \eta_S\} = \eta_S$. Using (7.17), (7.17) and letting $F_k^-(x) = \lim_{y \nearrow x} F_k(y)$, we obtain

$$F_S(\beta\eta_S) = 1 - \frac{\xi(V)}{V} (\eta_S - \beta\eta_L) \rightarrow 1 \quad \text{and} \quad F_S^-(\beta\eta_S) = 0 ,$$

$$F_L(\eta_S) = 1 \quad \text{and} \quad F_L^-(\eta_S) = \frac{\xi(V)}{V} (1 - \beta) \eta_S \rightarrow 0 .$$

These together imply that the probabilities of $b_S = \beta\eta_S$ and $b_L = \eta_S$ tend to 1. Consequently, the probability $b_S + b_L = (1 + \beta)\eta_S$ tends to 1 as $V \rightarrow \bar{V}$ from below. This establishes the statement for V being smaller but sufficiently close to \bar{V} for Proposition 4.

Appendix D: Omitted materials for the empirical analyses

D.1. Positive Serial Correlation and Stationarity

We first provide evidence that the time series of voter turnout exhibit strong positive serial correlation and are stationary. We model this by assuming that β_c follows an AR(1) process. If we assume that there is no tipping, that is $g = 0$ or $Q_1 \in \{0, 1\}$ this means that turnout is also an AR(1) process which we may write as

$$\tau_t = \rho_0 + \rho_1 \tau_{t-1} + \varepsilon_t$$

with $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ i.i.d and independent of τ_t . We can then estimate $\hat{\rho}_0$, $\hat{\rho}_1$ and $\hat{\sigma}_\varepsilon^2$ using standard OLS. The stationary distribution for τ_t is then a normal distribution with mean $\hat{\mu}_{stationary} = \frac{\hat{\rho}_0}{1-\hat{\rho}_1}$ and variance $\hat{\sigma}_{stationary}^2 = \frac{\hat{\sigma}_\varepsilon^2}{1-\hat{\rho}_1^2}$. Estimation results are reported in Table 2. For both US and UK data, the estimated $\hat{\rho}_1$'s are positive and statistically significant.³⁰

Table 2: OLS estimation results for the AR(1) models

	$\hat{\rho}_0$	$\hat{\rho}_1$	$\hat{\sigma}_\varepsilon$	$\hat{\mu}_{stationary}$	$\hat{\sigma}_{stationary}$
US data	0.314 (0.091)	0.439 (0.165)	0.037	0.560	0.041
UK data	0.349 (0.131)	0.526 (0.172)	0.046	0.737	0.054

Note: Newey-West standard errors robust to heteroskedasticity and first-order autocorrelation are reported in parentheses below the estimates $\hat{\rho}_0$ and $\hat{\rho}_1$.

Moreover, augmented Dickey-Fuller tests reject unit root hypotheses (i.e., $\rho_1 = 1$) for both US and UK turnout data.³¹ These results indicate that the time series of voter turnout exhibit strong positive serial correlation and are stationary.

D.2. The Monte-Carlo method for sampling error

We simulate $M = 10000$ samples drawn from the serially correlated model without tipping from the AR(1) model estimated above. Each simulated sample m consists of a sequence $\{\tau_t^m\}_{t=1}^T$ of observations. For each sample m the initial observation τ_1^m is simulated from the stationary distribution $\mathcal{N}(\hat{\mu}_{stationary}, \hat{\sigma}_{stationary}^2)$. For all $t > 1$, observations τ_t^m are simulated using

$$\tau_t^m = \hat{\rho}_0 + \hat{\rho}_1 \tau_{t-1}^m + \varepsilon_t^m,$$

where coefficients $\hat{\rho}_0$ and $\hat{\rho}_1$ are taken from Table 2, and each ε_t^m is drawn independently from $\mathcal{N}(0, \hat{\sigma}_\varepsilon^2)$. For each simulated sample sequence $\{\tau_t^m\}_{t=1}^T$ we estimate the parameter vector $\hat{\vartheta}_m$, where $m = 1, \dots, 10000$. This yields a collection of estimates $\{\hat{\vartheta}_m\}_{m=1}^{10000}$. The empirical cumulative distributions of $\{\hat{g}_m\}_{m=1}^{10000}$ and $\{\hat{\sigma}_{B,m}\}_{m=1}^{10000}$ from Monte Carlo simulations for both UK and US data are presented in Figure 7.2.

Table 3 reports the probability that \hat{g}_m or $\hat{\sigma}_{B,m}$ obtained from the empirical distributions of these estimates would generate values as large as those observed in the actual data.

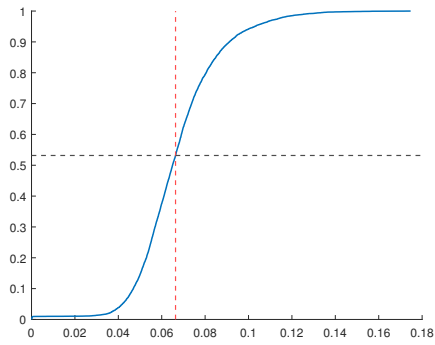
We should note that the procedure used here is conceptually the same as a randomization or permutation test (Young, 2019) in the sense that we ask how likely it is under the null hypothesis that we would see coefficient estimates as high as those we estimated: we do

³⁰ p values of two-sided tests for $\rho_1 = 0$ are 0.014 for US and 0.007 for UK.

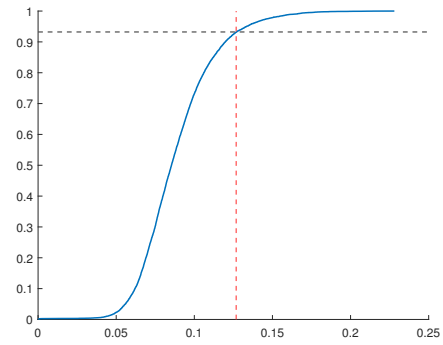
³¹ MacKinnon approximate p values are 0.0383 for US presidential elections and 0.0192 for UK general elections. Both tests reject the null unit root hypotheses at 5% significance level.

Figure 7.2: Empirical distribution of estimates \hat{g}_m and $\hat{\sigma}_{B,m}$ from Monte Carlo experiments

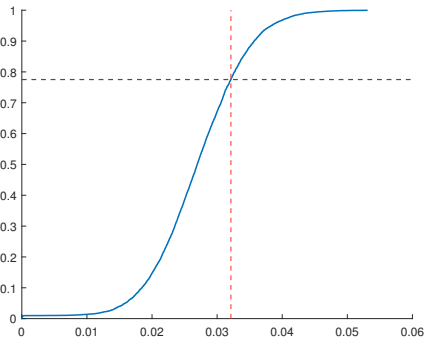
(a) Empirical CDF of estimated \hat{g}_m for US



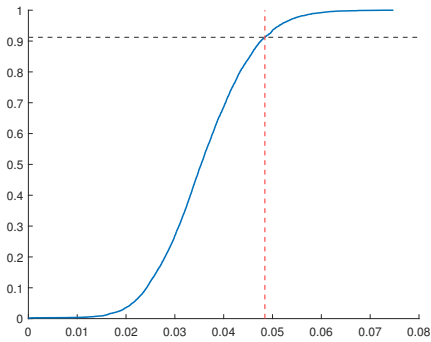
(b) Empirical CDF of estimated \hat{g}_m for UK



(c) Empirical CDF of estimated $\hat{\sigma}_{B,m}$ for US



(d) Empirical CDF of estimated $\hat{\sigma}_{B,m}$ for UK



Note: In all these figures, the red dashed lines denote the estimates obtained from the real data, and the black dashed lines denote probability of observing estimates that are equal or lower than the estimates obtained from real data.

Table 3: Probabilities of data or higher from Monte Carlo experiments

	$\% [\hat{g}_m \geq \hat{g}]$	$\% [\hat{\sigma}_{B,m} \geq \hat{\sigma}_B]$
UK data	0.068	0.088
US data	0.468	0.225

not ask the t -test question of how likely it is under the null hypothesis that we would see the ratio of coefficient estimates to standard errors that we see in the data. The reason for this is simple, the latter question is without economic interest: we are not concerned with whether the null hypothesis is exactly true, we know *a priori* it is not. In particular if we observe a low value of the gap, say 2% we would conclude that tipping was not an important phenomenon and would reject it as a useful model no matter the precision with which the coefficient of 2% was estimated. By contrast a t -test would not reject the null hypothesis if the standard error was sufficiently small. In other words we do not use a t -test approach because it is without useful economic meaning.