Polarization and Electoral Balance☆

David K. Levine[1], Andrea Mattozzi[2]

Abstract

We study a model of electoral competition in which two politicians with different office motivations set party platforms and both politicians and grassroots can provide electoral effort. While the underlying structure of the model is asymmetric, we show that both parties have an equal chance of winning the election. In equilibrium, however, only the most office motivated politican matters for policy polarization and welfare: a kind of Gresham's law for politicians. The greater this office motivation, the greater is polarization and the lower is welfare. Less interest in politics means also greater polarization and lower welfare.

JEL Classification Numbers: D72

Keywords: Elections, Politicians, Grassroots, Polarization

☆We are especially grateful to Giacomo Calzolari, Rohan Dutta, Ennio Flaiano, Gabriele Gratton, Cesar Martinelli, and Salvatore Modica. Konuray Mutluer provided excellent research assistance. We gratefully acknowledge support from MIUR PRIN 2017 n. 2017H5KPLL_01.

[1]Department of Economics, EUI and WUSTL, david@dklevine.com

[2]Department of Economics, University of Bologna, andrea.mattozzi@unibo.it

1. Introduction

The history of U.S. national elections and political parties is marked by a number of intertwined stylized facts, which are rather intriguing from a political economy perspective. First, the outcomes of U.S. national elections have always been evenly balanced between parties. During the nearly 200 years since the formation of the Democratic party in 1828 a Democrat has won the presidential election roughly half the time.[3] The second fact is indicative of the mechanism through which the two parties have maintained parity over the years. In the early 1960s the Democratic party began to "steal" the issue of civil rights from the Republicans: by doing so they built a supermajority of more than 2/3rds in the U.S. Senate in the 89th Congress (1963-1965).[4] The Republicans responded with their "Southern Strategy" of changing positions on civil rights and building a grassroots anti-civil rights movement in the South, successfully convincing Strom Thurmond to switch parties in 1964, and resulting today in the Republicans being the party of the South and the Democrats the party of the North, a reversal of their position prior to the 1960s. Third and more generally, there has always been a substantial difference between the policies of the two parties: the enormous historical gap between Democrats and Republicans over civil rights being just one of many examples. Furthermore, the degree of polarization between the parties has varied substantially over time.[5] Finally since the "Era of Good Feelings" which ended about the time the Democratic party was formed, there has never been a centrist party.

Strong empirical regularities call out for a strong explanation, and the obvious explanation of parity between the two parties is that party platforms adjust so as to give both parties an equal chance.[6] This would be the prediction of a

[3]For smaller jurisdictions in the US, party affiliations seem to be determined by preferences over national issues, so we do not expect parity there.

[4]Since 1828 such division in the Senate has happened only in 1837, during the Civil War and reconstruction era from 1861 to 1875, and during the great depression era from 1935-1943. See https://www.senate.gov/history/partydiv.htm.

[5]See for example Hall (2019).

[6]Note that the issue here is the frequency with which the two parties win elections and, not as in Levine and Martinelli (2022) why elections may be close.

standard Downsian model - the Hotelling model applied to politics. This model also captures the fact that a party that tries to "steal" issues from the other it will reverse positions. Unfortunately, the Downsian model is inconsistent with the fact that there is a substantial difference in party policies and so it is not suited to explain polarization. Our goal in this paper is to build a model that delivers the aforementioned stylized facts and it does so without any underlying symmetries in the preferences of voters or politicians.

The key ingredient in our model is that in addition to office motivated politicians and policy concerned voters there are ideologically motivated grassroots who are crucial for mobilizing voters: indeed, it was the ability of the Republican party to recruit these grassroots that was the key to the success of their southern strategy.[7] Specifically, we view the political process as taking place in three stages. First, politicians establish platforms. These are two dimensional: the platform specifies the type of policies that will be implements but also the intensity with which these policies will be pursued. The type of policy measures the stands on the issues: for or against abortion, gun control, taxes, goverment spending, immigration and so forth. By contrast the intensity of policy measures the extent to which the policies will be implemented: it may be that few if any laws or other changes are proposed, or it may be that extensive effort are proposed to implement policies. Hence, for example Ronald Reagan's "big tent" promised a right wing agenda, but with low intensity. In the second stage, grassroots and voters choose which platform to support. Finally, the politicians and grassroots compete to mobilize voters.

In this model polarization plays a key role: by intensely pursuing different platforms grassroots are highly motivated to mobilize voters and this reduces the cost to politicians of doing so. The striking conclusion is that the degree of polarization is determined by the most office motivated politician - a kind of Gresham's law for politicians. The greater this office motivation, the greater is polarization and the lower is welfare.

[7]Our model captures in the simplest way the fact that political parties are typically made up of several layers with leaders at the top, grassroots organizers and turnout brokers in the middle, and voters at the bottom.

A novel feature our approach is a variation on the Downsian model that allows for policy differences. Rather than viewing a policy as a single point we view policy as a random outcome relative to a policy target. This recognizes the reality that the issues that will be salient or important are not known at the time of the campaign, so that a policy platform is merely indicative of what sorts of policies will be followed. In this way, we can still distinguish clearly between left and right parties. For example, the support of a left party platform will be bounded above but not below: depending on circumstances it may pursue extreme left policies, but it will never pursue extreme right policies and vice versa for a right party platform.

Our theory of grassroots motivation also explains why there are no centrist parties: it is the combination of the certainty of no extreme disliked policies and the possibility of extreme liked policies that attracts grassroots. A centrist party that has unbounded support on both sides does not rule out the possibility of extreme disliked policies and one that has bounded support on both sides rules out the possibility of extreme liked policies. Hence centrist parties do not attract grassroots and without grassroots cannot survive.

Our novel modelling of policy platforms allows for the southern strategy - after observing the targets, a politician may pivot to the left or right. The policy target in this interpretation represents a pivot point: by choosing the target the politician indicates willingness to pivot to either left or right from that point. As in the Downsian model equilibrium forces the policy targets into alignment, but unlike the Downsian model this leaves parties that are distinctly different.

As indicated, platforms have two dimensions: policy limits and the intensity of that policy. In our view both contribute to polarization. To take an example: the Republican party has historically been anti-abortion and the Democratic party pro-choice - these are substantially different positions. A debate over particular medical procedures in the third trimester of pregnancy we would regard involving low intensity and leading to relatively little polarization, while a debate over whether abortion should be unrestricted or banned under all circumstances we regard as having high intensity and a high degree of polarization.

We emphasize that a unique element of our model is the idea that a subset of voters are not simply driven like sheep by politicians but engage in self-organization: the grassroots matter. Perhaps nowhere was this more apparent than in the 2016 US Presidential election where the Democratic party was blind-sided by grass-roots organizations that got out the vote for Donald Trump in the absence of much Republican party effort. In the subsequent Congressional election this effort was matched, despite all gerrymandering, by an even more striking Democratic grass-roots movement. The history of politics is full of grassroots movements ranging from labor unions to social clubs and just as party leaders put effort into turning out the vote so do the grassroots. There are many existing models of platform competition and while all have a role for both politicians and voters, few have a role for grassroots.[8] In addition, in our model voters' turnout is a function of both policy platforms and electoral effort. In light of the increasing importance of GOTV campaigns in political elections,[9] our grassroots model provides an empirically grounded alternative to the valence competition models that follow Stokes (1963) critique to the Downsian model.[10]

In models such as Herrera, Levine and Martinelli (2008), Callander (2008), and Hirsch (2022) politicians have both office motivation and ideological motivation. In particular Callander (2008) uses a signaling model very unlike the model here but does have a result similar to ours: office motivated candidates drive out policy oriented candidates. To explain balanced elections, however, such models must assume that political parties are symmetric - a fact that in a sense is the one that needs to be explained. We assume instead that politicians are purely office motivated and place the ideological motivation with the grass-

[8]Notable exceptions are Schofield (2006), Miller and Schofield (2007) and Venkatesh (2020). None of these works, however, considers substitutability between candidates and grassroots effort and its effect on polarization.

[9]See Green and Gerber (2019) for voting mobilization and grassroots movements in US. Enos, Fowler and Vavreck (2013) provide evidence that GOTV, by increasing the differences between voters and nonvoters, may lead to an increase in political inequality.

[10]See Carillo and Castanheira (2008), Eyster and Kittsteiner (2007) and Meirowitz (2007) for models with "endogenous" valence and Groseclose (2001), Aragones and Palfrey (2002), and Kartik and McAfee (2007) for the case of exogenous valence advantage. A review of this literature can be found in Iaryczower and Mattozzi (2013).

roots. There is a great deal of evidence for this. The triangulation of Clinton and Blair was widely criticized as opportunistic. George H. W. Bush conveniently switched positions on abortion when it advanced his political career, Boris Johnson thought that Brexit was a terrible idea before he was for it, and of course Trump was a liberal New York Democrat before becoming a conservative Republican. A related model assuming purely office motivated politicians in the valence competition literature is Ashworth and Bueno De Mesquita (2009). Their model, however, is a symmetric one in which politicians have an incentive for more polarized platforms because it causes a volatile electorate to focus on issues rather than valence competition, and so increases rents to politicians. We focus instead on the substitutability between the campaign effort of candidates and grassroots to turnout voters in the context of a simple asymmetric contest that leads to simple and sharp results concerning equilibrium.

2. The Model

We study a multi-stage political contest. There are two politicians $j = \{L, R\}$, two representative grassroots[11] $k \in \{\ell, r\}$, and a mass of potential voters. The contest has three stages. First, politicians choose their campaign platforms. Second, voters choose their political affiliations. Finally politicians and grassroots choose electoral effort, which mobilizes voters and determines the outcome of the election.

In the first stage politicians each choose a *policy target* $q_j \in \Re$. Observing the policy target of their opponent they then determine whether to purse a *left* or *right* platform, also labeled by $k \in \{\ell, r\}$. This provides a simple way in which politicians can pivot against an opponent who tries to "steal" their voters. The actual policy implemented by the winning party is the combination of a deterministic component, the policy target, and a random shock. The random shock depends upon unpredictable circumstances, such as wars or recessions, after the election: for a left platform this is given by a square integrable continuous random variable P_ℓ with support bounded above but not below, and for a

[11] These might also be interest groups.

right platform is given by P_r with support bounded below but not above. Furthermore, we assume that $\mathrm{E}[P_\ell] < \mathrm{E}[P_r]$. Hence, for a winning left platform of politician j the final policy implemented is $q_j + P_\ell$ and for a winning right platform it is $q_j + P_r$. The random shocks define what it means for a platform to be left or right: a left platform will only implement policies below a threshold and a right platform only policies above a threshold.[12] After these policy distributions are established, politicians choose the second dimension of their platform: the *intensity* of their platform $x_j \geq 0$. This measures how aggressively the finally chosen policy will be pursued.

In the second stage voters choose to *affiliate* with platforms. This results in a mass $z_j \geq 0$ supporting the platform of politician j with $z_L + z_R = 1$. If a voter is mobilized they vote for the platform they support.

In the third stage politicians first choose their electoral effort $e_j \geq 0$, followed by grassroots choosing their electoral effort $E_k \geq 0$. Efforts mobilize voters depending on the platform they support and we assume that effort of politicians and grassroots are perfect substitutes.[13] If politican j has platform k the number of voters mobilized is $\lambda_k(e_j + E_k)$. Here $\lambda_k > 0$ captures the *efficacy* of mobilization efforts.[14] This efficacy is greater the more voters support the platform.[15] Specifically, the efficacy of effort for platform j is $\lambda_k = H_k(z_k) > 0$ a strictly increasing continuous function of the mass of supporters. We assume

[12]These shocks may also be due to the views of different politicians in the party. In the current US Senate the most right wing Democrats, Sinema and Manchin, have strong influence due to the close division of the parties. If, for example, a Republican Senator died and was replaced on an interim basis by a Democrat, their influence would be greatly mitigated. Notice that the most right wing Democrats are close in policy positions to the most left wing Republicans, Collins and Murkowski, yet nobody would claim that the US Senate is not polarized.

[13]This greatly simplifies computations because the marginal cost of effort by the grassroots does not depend upon the effort of the politician. We indicate below that the basic ideas carry over with imperfect substitutes.

[14]Our modelling of voters' mobilization follows standard assumptions adopted by the existing group-turnout models. See Levine and Mattozzi (2020) for a recent formal model of voter mobilization and a review of the literature. In addition, and differently from existing models, here we assume that policy affects mobilization too.

[15]For lobbying effort it may be the other way around, but Levine and Mattozzi (2020) show that in important elections larger parties have lower costs of turning out voters.

that for some intermediate population the efficacies are the same, that is, for some $0 < z^* < 1$ we have $H_\ell(z^*) = H_r(1 - z^*)$.

The platform (and politician) who mobilizes the most voters wins the election. In case of a tie we utilize an endogenous tie-breaking rule - so, for example, if one platform can provide no effort while the other can, then to reflect the fact that it is possible to win by providing a miniscule amount of effort, we model this as a tie in which the platform that can provide effort wins.

The choices of politicians, grassroots, and voters depend upon their preferences. The politician of party j receives a reward $B_j > 0$ for winning and nothing for losing. This office motivation represents how much rent the politician expects to get from the office, either from power or from money. In addition, effort is costly. Letting p_j be the probability that politician j wins, the expected utility of a politician is $u_j = p_j B_j - e_j$.

Voters preferences are determined by their ideal points y distributed according to a continuous strictly increasing distribution G over the real line. If the realized value of the winning platform is $\tilde{q}$ a voter with ideal point y receives utility $-|y - \tilde{q}|^2$. That is, policy is measured in units so that voter utility is quadratic in distance from the ideal point, or equivalently, all voters have the same constant absolute risk aversion. Voter expected utility from a platform is then calculated from the distribution of policies that will occur if that platform wins, and they affiliate with the platform whose policy distribution gives the highest expected utility.

Grassroots are motivated by the intensity with which they feel suitable platforms will implemented. For simplicity we describe the preferences of ℓ grassroots, with the preferences of r grassroots symmetrically defined. The ℓ grassroots find a platform suitable and will help to mobilize voters for it if they find the supporters simpatico and if the platform is left-wing. There are three conditions for a platform j to be suitable for grassroots. First, there must be a threshold μ such that all voters with $y < \mu$ are willing to affiliate with platform j. Second, platform j must be a left platform, that is, offer policies that are bounded above and unbounded below. Finally, if there are two identical left platforms only the one led by politician $j = L$ is suitable. Consequently there is either one suitable platform or none. In this context we say that a platform has *grassroots support* if it is suitable. If a platform is not suitable the grassroots receive no utility from supporting it. If platform k is suitable, ℓ gets a utility of x_k for winning and $-x_{-k}$ for losing. In addition effort provision is costly: letting p_k be the probability that platform k wins, the expected utility of grassroots ℓ is $v_\ell = p_k x_k - (1-p_k)x_{-k} - E_\ell$.

Letting $\tilde{q}_k$ be the realized value of platform k we may measure *polarization* by the expected value V of $(x_k + x_{-k})|\tilde{q}_k - \tilde{q}_{-k}|$, that is the expected product of the strength of policies times the difference in policies. This captures the idea that polarization measures the intensity of conflict. If the two platforms have little difference in policies $|\tilde{q}_k - \tilde{q}_k|$ there is little polarization, but even if they have a large difference, if these policies are not aggressively pursued because $x_k + x_{-k}$ is small, there is also little polarization.

Because policy is uncertain and voters are risk averse, they care both about positions as measured by $q_j + \mathrm{E}[P_j]$ but also about risk as measured by $\mathrm{var}[P_j]$: a platform with a less good position may be preferred because it is less risky. We are interested, however, in elections where it is differences in policy positions that matter rather than differences in risk. Hence, while we do not assume that both parties are equally risky, we do make a technical assumption that differences in risk are not too great compared to differences in positions: specifically we assume that

$$A1)\, |\mathrm{var}[P_\ell] - \mathrm{var}[P_r]| < \left[\mathrm{E}[P_r] - \mathrm{E}[P_\ell]\right]^2/2.$$

Our solution concept is subgame perfect equilibrium.

3. The Equilibrium

Define the most office motivated politician $w \in \{L, R\}$ to be such that $B_w \geq B_{-w}$, that is the one who gets the greatest rent from office-holding. Note that if there is a tie both politicians are by definition most office motivated. By a *least polarized equilibrium* we mean any equilibrium with the least polarization V in the set of equilibria. Define $\Delta = \mathrm{E}[P_r] - \mathrm{E}[P_\ell] > 0$.

Theorem 1. *(i) There is a unique value q^* such that in any equilibrium $q_w = q_{-w} = q^*$, politician L chooses the ℓ platform, R chooses r and both parties have an equal chance of winning. Neither politician k provides any effort, and each gets expected utility $B_k/2$.*

(ii) With probability one polarization is at least $V \geq \Delta B_w/2$ and grassroots welfare is the expected value of $-V/2$.

(iii) There is a least polarized equilibrium with $V = \Delta B_w/2$. All least polarized equilibria have pure strategy platforms with $x_w \leq B_{-w}$, $x_{-w} \leq B_w/2$.

The theorem contains a number of insights.

First, it is an Hotelling type of result: policy limits are uniquely chosen to equalize the efficacy of effort. While the endogenous division of support in the population does not need to be symmetric in equilibrim, still each party has an equal chance of winning the election.

Second, politicians choose not to provide effort. While effort by grassroots and politicians are substitutes, polarization and grassroots effort are strategic complements: politicians create polarization to motivate voters and avoid providing their own effort.

Third, the utility of grassroots is proportional to the negative of polarization while the politicians and voters are indifferent. Hence we may unambiguously identify greater polarization with lower welfare. This gives particular meaning to least polarized equilibria as these are exactly the welfare maximizing equilibria.[16]

Fourth, turning to least polarized equilibria, only the most office motivated politician matters for polarization and the greater his office motivation, the higher polarization. Intuitively, the politician who has the most to gain from winning will be the one with stronger incentives to exploit polarization in order to better motivate the grassroots in mobilizing voters. This mechanism will be

[16] In fact politicians may also be mildly averse to polarization. First, they are concerned with their legacy - highly polarizing politicians are less likely to be remembered fondly. In a similar vein less polarization means that the opposition is less likely to engage in character assassination. Real assassination could be also an issue: both the Lincoln and McKinley assassinations appear to have been motivated in part by the highly polarized political atmosphere. The recent attack on the US Congress is another example of violence against politicians that arose from political polarization.

the stronger, the less the grassroots care about issues.

Finally, turning from welfare to distribution, we observe $x_w \leq B_{-w}$, that is lower values of B_{-w} restrict the ability of the most office motivated politician w to increase the intensity of their policy platform. This is bad for their own grassroots and voters and good for the grassroots and voters of the other politician. That is to say, if the most office motivated politician chooses to maximize the strenght of their policy platform (that is they takes an extreme policy position), grassroots and voters led by the other politician are better off the less office motivated their own leader is: the race to dissipate rents will be less fierce. Similarly lower values of B_w restrict the possibility of taking extreme positions of the less office motivated politician.

4. Analysis of the Game

In analyzing the game step-by-step starting at the end it will be useful to introduce the terminology of *reduced games*: these are the family of games played at a particular step where each outcome of the game is assigned an equilibrium from the remaining future subgame. Hence we look for Nash equilibria of these reduced games.

4.1. Platform Affiliation

It is convenient first to analyze the choice of voters as this does not depend on any subsequent events in later stages of the game, and to analyze which platforms get grassroots support as this depends only on the choice of voters and not on subsequent events in the game. For $q_r + \mathrm{E}[P_r] \neq q_\ell + \mathrm{E}[P_\ell]$ define

$$\hat{y} = \frac{(q_r + \mathrm{E}[P_r]) + (q_\ell + \mathrm{E}[P_\ell])}{2} + \frac{\mathrm{var}[P_\ell] - \mathrm{var}[P_r]}{(q_r + \mathrm{E}[P_r]) - (q_\ell + \mathrm{E}[P_\ell])}.$$

Proposition 1. *Case (i) If there are two platforms leaning ℓ then there is μ such that all voters with $y < \mu$ affiliate with the platform with $q_j < q_{-j}$. The reverse holds if there are two platforms leaning r.*

Case (ii) If there is both a left and a right platform and $q_r + E[P_r] < q_\ell + E[P_\ell]$ then voters with $y < \hat{y}$ affiliate with the right platform and those with $y > \hat{y}$

affiliate with the left platform. If $q_r+E[P_r] > q_\ell+E[P_\ell]$ voters with $y < \hat{y}$ affiliate with the left platform and those with $y > \hat{y}$ affiliate with the right platform. If $q_r + E[P_r] = q_\ell + E[P_\ell]$ and $\mathrm{var}[P_j] < \mathrm{var}[P_{-j}]$ then all voters affiliate with platform j, while if $\mathrm{var}[P_j] = \mathrm{var}[P_{-j}]$ voters are completely indifferent between the platforms.

Under the technical assumption A1, for $q_r \geq q_\ell$ we have $\hat{y}$ decreasing in q_r and increasing in q_ℓ.

Proof. We can write the expected utility of voter y for platform k as

$$-\mathrm{E}[|y - (q_k + P_k)|^2] =$$

$$-\left(y^2 - 2y(q_k + \mathrm{E}[P_k]) + q_k^2 + 2q_k\mathrm{E}[P_k] + \mathrm{E}[P^2] + (\mathrm{E}[P_k])^2 - (\mathrm{E}[P_k])^2\right) =$$

$$-\left(y^2 - 2y(q_k + \mathrm{E}[P_k]) + (q_k + \mathrm{E}[P_k])^2 + \mathrm{var}[P_k]\right).$$

Hence the utility advantage of r over ℓ is given by

$$2y(q_r+\mathrm{E}[P_r]) - (q_r + \mathrm{E}[P_r])^2 - 2y(q_\ell+\mathrm{E}[P_\ell]) + (q_\ell + \mathrm{E}[P_\ell])^2 - \mathrm{var}[P_r] + \mathrm{var}[P_\ell] =$$

$$2y\left[(q_r + \mathrm{E}[P_r]) - (q_\ell + \mathrm{E}[P_\ell])\right] - (q_r + \mathrm{E}[P_r])^2 + (q_\ell + \mathrm{E}[P_\ell])^2 - \mathrm{var}[P_r] + \mathrm{var}[P_\ell].$$

If $q_r+\mathrm{E}[P_r] \neq q_\ell+\mathrm{E}[P_\ell]$, the above expression equals zero at $\hat{y}$ and is decreasing in y for $q_r + \mathrm{E}[P_r] < q_\ell + \mathrm{E}[P_\ell]$ and increasing in y for $q_r + \mathrm{E}[P_r] > q_\ell + \mathrm{E}[P_\ell]$. This gives the first result.

If $q_r + \mathrm{E}[P_r] = q_\ell + \mathrm{E}[P_\ell]$ then y does not matter for voter choice and the second result follows from

$$-(q_r + \mathrm{E}[P_r])^2 + (q_\ell + \mathrm{E}[P_\ell])^2 =$$

$$-\left[(q_r + \mathrm{E}[P_r]) - (q_\ell + \mathrm{E}[P_\ell])\right]\left[(q_r + \mathrm{E}[P_r]) + (q_\ell + \mathrm{E}[P_\ell])\right] = 0.$$

To show the use of the technical assumption $A1$, there is no loss of generality in showing that $\hat{y}$ is decreasing in q_r. Notice that if $q_r \geq q_\ell$ then certainly

$q_r + \mathrm{E}[P_r] > q_\ell + \mathrm{E}[P_\ell]$. Observe that $\hat{y}$ is decreasing in q_r if the derivative of

$$\frac{\mathrm{var}[P_\ell] - \mathrm{var}[P_r]}{(q_r + \mathrm{E}[P_r]) - (q_\ell + \mathrm{E}[P_\ell])}$$

is less than 1/2 in absolute value. The absolute value of the derivative is given by

$$\frac{|\mathrm{var}[P_\ell] - \mathrm{var}[P_r]|}{((q_r + \mathrm{E}[P_r]) - (q_\ell + \mathrm{E}[P_\ell]))^2}$$

and since $q_r \geq q_\ell$ by assumption, technical assumption $A1$ assures that this is strictly less than 1/2. □

Grassroots support is an immediate corollary.

Proposition 2. *Case (i) If there are two platforms leaning ℓ and $q_j < q_{-j}$, then grassroots ℓ supports the platform of politician j, and if $q_j = q_{-j}$, then grassroots ℓ supports platform $j = L$, while grassroots r support no platform. The reverse holds for two platforms leaning r.*

Case (ii) If there is both a left and a right platform and $q_r + E[P_r] < q_\ell + E[P_\ell]$ neither platform receives grassroots support. If $q_r + E[P_r] = q_\ell + E[P_\ell]$ and $\mathrm{var}[P_j] < \mathrm{var}[P_{-j}]$ platform j is supported by grassroots j and platform $-j$ receives no grassroots support. Otherwise platform k is supported by grassroots k.

4.2. Grassroots Effort Provision

We start with the final game between the grassroots. There are three cases depending on which platforms have grassroots support. The game is then defined by the efficacies of mobilization λ_ℓ, λ_r,by platform intensities x_ℓ, x_r and the effort provision of the politicians e_ℓ, e_r. In case both platforms have grassroots support one must be an ℓ and one an r platform so, abusing notation, we denote the left platform politician by ℓ and the right platform politician by r. Recall that polarization V is defined as the expected value of $(x_k + x_{-k})|\tilde{q}_k - \tilde{q}_{-k}|$. Further, we say that a platform is disadvantaged and we denote it by d if $\lambda_{-d}(V + e_{-d}) \geq \lambda_d(V + e_d)$.

Proposition 3. *(i) If neither platform has grassroots support (or $V = 0$) then the platform with $e_j > e_{-j}$ wins.*

(ii) If only platform j has grassroots support and $\lambda_j x_j > e_{-j}$ then platform j wins for certain.

(iii) If both platforms have grassroots support (and $V > 0$) either $V < (\lambda_{-k}/\lambda_{-k})e_{-k} - e_k$ and $p_k = 0$ or $0 < p_d < 1$. In the latter case, define $L = \lambda_d(V + e_d) - \max_k\{\lambda_{-k}e_{-k}, \lambda_k e_k\}$, then

$$p_d = (1/2)\left(2L/(V\lambda_{-d}) - (L/V)^2/(\lambda_{-d}\lambda_d)\right).$$

If $\lambda_k = \lambda_{-k}$ and $e_k = e_{-k}$ then the utility of grassroots is $v_k = -V/2$.

Proof. Case i. This immediate as the grassroots provide no effort.

Case ii. The value to the active grassroots of wining is $V \geq x_j$. The cost of outbidding e_{-j} is e_{-j}/λ_j. Hence the grassroots should guarantee a victory by bidding a miniscule amount more than e_{-j}.

Case iii. Notice that this is an all pay auction with complete information, linear costs, and (possibly) head starts. If platform k wins the election for certain, a grassroots k gets $x_k - E_k$ and if platform k loses for certain, grassroots k gets $-x_{-k} - E_k$. Hence the benefit of winning over losing is V, and k will provide this much effort to get a certain win over a certain loss. As usual in the all pay auction we think of the grassroots as "bidding" of a total amount of effort. It follows that the amount that k is willing to bid is $\lambda_k(V + e_k)$ since e_k units of effort are provided by the politician for free and the efficacy of effort is λ_k. Hence for $\lambda_{-k}e_{-k} \geq \lambda_k e_k$ in this linear all-pay auction $-k$ has a head-start advantage of $\lambda_{-k}e_{-k} - \lambda_k e_k$.

Recall that at least one party d is disadvantaged, that is, satisfies $\lambda_{-d}(V + e_{-d}) \geq \lambda_d(V + e_d)$. From standard results on complete information all-pay auctions (see for example Hillman and Riley (2006) and Baye, Kovenock and De Vries (1998)) there is a unique equilibrium, both grassroots adopt mixed strategies with a known structure.[17] The lowest bid of $-k$ is $\lambda_{-k}e_{-k}$ and the

[17] See Levine and Mattozzi (2020) for details.

most k is willing to bid is $\lambda_k(V+e_k)$. Hence if $\lambda_{-k}e_{-k} > \lambda_k(V+e_k)$ it follows that k loses for certain. This gives the first result concerning p_k. Otherwise define $\underline{b} = \max_k\{\lambda_{-k}e_{-k}, \lambda_k e_k\}$. On $(\underline{b}, \lambda_d(V+e_d)]$ grassroots k plays a uniform mixture with density height f_k, and has an atom at $\lambda_k e_k$ of F_k. This atom is calibrated so as to make the opponent indifferent between low and high bids.

The values of f_k, F_k are easily computed. A bid of $b_k > \underline{b}$ wins V with probability $f_{-k}(b_k - \underline{b})$ and costs b_k/λ_k with probability 1. Indifference of k then implies that $f_{-k} = 1/(V\lambda_k)$. It follows that if we define $L = \lambda_d(V+e_d) - \underline{b}$ then $F_k = 1 - f_k L = 1 - L/(V\lambda_{-k})$.

Note that if d has the headstart advantage so $\lambda_d e_d \geq \lambda_{-d}e_{-d}$ then $L = \lambda_d V$ so $F_{-d} = 1 - L/(V\lambda_d) = 0$ and $-d$ has no atom in this case. From this we may compute the probability of d winning. With probability F_d the disadvantaged grassroots opts out and loses for sure. Otherwise with probability $1 - F_d$ with probability F_{-d} the advantaged grassroots opts out and the disadvantaged grassroots wins for sure, or with probability $1 - F_{-d}$ both bid uniformly on $(\underline{b}, \lambda_d(V+e_d)]$, implying that each has a 50% chance of winning. Note in particular that since $F_d < 1$ we have $0 < p_d < 1$. More specifically,

$$p_d = (1-F_d)\left(F_{-d} + (1/2)(1-F_{-d})\right) =$$

$$(1/2)(1-F_d)\left(1+F_{-d}\right) =$$

$$(1/2)(L/(V\lambda_{-d}))\left(2 - L/(V\lambda_d)\right) =$$

$$(1/2)\left(2L/(V\lambda_{-d}) - (L/V)^2/(\lambda_{-d}\lambda_d)\right).$$

Finally, if $\lambda_k = \lambda_{-k}$ and $e_{-k} = e_k$ the contest is symmetric so there is complete rent dissipation so that $v_k = V/2$. □

4.3. Politician Effort Provision

We continue with the reduced effort provision game between the politicians. Again there are three cases depending on which platforms have grassroots support. The game is defined by the efficacy of mobilization λ_ℓ, λ_r and by the platform intensity x_ℓ, x_r. We define an equilibrium of the effort provision game between politicians as *peaceful* if $e_k = e_{-k} = 0$ with probability one. Otherwise we call the equilibrium *contested.* Define $\overline{V} \equiv (\max_k \lambda_k / \min_k \lambda_k) \max B_k$

Proposition 4. *(i) If neither platform has grassroots support then $p_L, p_R > 0$.*

(ii) If only platform j has grassroots support and $V > \overline{V}$ then platform j wins for sure.

(iii) If both platforms have grassroots support and $V > \overline{V}$, there is a unique Nash equilibrium of the politician effort provision subgame and it is peaceful with each politician having a positive chance of winning.

The computations in the proof use the fact that effort by grassroots and politicians are perfect substitutes. However, the key idea is that greater polarization causes the grassroots to make greater effort, and so it reduces the marginal benefit of politician effort: this idea remains valid even without perfect substitutes. Hence, when there is "enough" polarization as exactly computed here, politicians choose not to provide effort.

Proof. Case i. This is now an all-pay auction between the two politicians without headstart and with prizes $B_j > 0$ and effort efficacy $\lambda_j > 0$. The result that each has a positive chance of winning is standard.

Case ii. Follows directly from Proposition 3.

Case iii. Let G_k, G_{-k} denote the equilibrium strategies for the politicians choice of effort in the third stage of the game contingent on the earlier platform choices, $u_k(G_k, G_{-k})$ their expected utility and $p(G_k, G_{-k})$ the probability of winning.

We may assume $e_k \leq B_k$ since it could not be optimal to bid more than the value of the prize. We may assume without loss of generality that $V > (\lambda_{-k}/\lambda_{-k})B_w$. By Proposition 3 this implies that for $\lambda_{-d}(V + e_{-d}) \geq \lambda_d(V + e_d)$, $L = \lambda_d(V + e_d) - \max_k\{\lambda_{-k}e_{-k}, \lambda_k e_k\}$ and

$$p_d = (1/2)\left(2L/(V\lambda_{-d}) - (L/V)^2/(\lambda_{-d}\lambda_d)\right).$$

From this we may compute the derivative

$$\frac{\partial p_d}{\partial L} = (1/V)\left(1/\lambda_{-d} - (L/V)/(\lambda_{-d}\lambda_d)\right).$$

Since $L \le \max_k \lambda_k V$ we have

$$-(1/V)\left(\max_k \lambda_k\right)/(\lambda_{-d}\lambda_d) \le \frac{\partial p_d}{\partial L} \le (1/V)\,(1/\lambda_{-d})\,.$$

Moreover $-\max_k \lambda_k \le \partial L/\partial e_k \le \lambda_d$, so

$$-(1/(AV))\max_k \lambda_k\left(\max_k \lambda_k\right)/(\lambda_{-d}\lambda_d) \le \frac{\partial p_d}{\partial e_k} \le (1/V)\max_k \lambda_k/\lambda_{-d}.$$

From this it follows that

$$(1/V)\max_k \lambda_k\left(\max_k \lambda_k\right)/(\lambda_{-d}\lambda_d) \ge \frac{\partial p_{-d}}{\partial e_k} \ge -(1/V)\max_k \lambda_k/\lambda_{-d}.$$

Summarizing, for any λ_d, λ_{-d} we have $\partial p_k/\partial e_k \le (\max_k \lambda_k/\min_k \lambda_k)/V$.

The expected utility for politician k for e_k is $u_k(e_k, G_{-k}) = B_k \int p(e_k, G_{-k})dG_{-k} - e_k$. Differentiating under the integral sign we get

$$\frac{\partial u_k}{\partial e_k} = B_k \int \frac{\partial p(e_k, G_{-k})}{\partial e_k} dG_{-k} - 1 \le B_w\left(\max_k \lambda_k/\min_k \lambda_k\right)/V - 1$$

so that for $V > (\max_k \lambda_k/\min_k \lambda_k)\,B_w$ the only equilibrium is for both politicians to choose $e_k = 0$. From Proposition 3 this also implies that each politician has a positive chance of winning. □

4.4. Platform Intensity

The platform intensity reduced has the following properties

Proposition 5. *(i) If neither platform has grassroots support then intensity does not matter.*

(ii) If only platform j has grassroots support then equilibrium in the politician effort provision reduced game is peaceful and j wins for sure.

(iii) If both platforms have grassroots support and $q_r + E[P_r] > q_\ell - E[P_\ell]$ then equilibrium in the politician effort provision reduced game is peaceful and each politician has a positive chance of winning.

Proof. Case i. Obvious.

Case ii. Politician j needs merely to choose $x_j > \overline{V}$ to assure this result.

Case iii. Let $\overline{e}_k$ be the expected effort in some Nash equilibrium of the politician effort reduced game, and let $\hat{p}_k, \hat{p}_{-k}$ be the winning probabilities. From Proposition 4 k can guarantee a peaceful equilibrium by choosing $x_k \geq \overline{V}$. By Proposition 3 this results in a unique equilibrium of the grassroots effort reduced game with winning probabilities $p_k, p_{-k} > 0$. Hence $\hat{p}_k B_k - \overline{e}_k \geq p_k B_k$ and $(1 - \hat{p}_k) B_{-k} - \overline{e}_{-k} \geq (1 - p_k) B_{-k}$. Dividing the first inequality by B_k and the second by B_{-k} and adding we see that

$$1 - \overline{e}_k / B_k - \overline{e}_{-k} / B_{-k} \geq 1.$$

This is possible if and only if $\overline{e}_k = \overline{e}_{-k} = 0$, which is to say equilibrium of the politician effort reduced game is peaceful. □

4.5. Platform Direction

Proposition 6. *The platform direction reduced game has a unique equilibrium: if $q_j < q_{-j}$ then politician j chooses a left platform and politician $-j$ a right platform; if $q_j = q_{-j}$ then politician L chooses the ℓ platform and politician R chooses the r platform.*

Proof. For each pair q_j, q_{-j} each politician j has two strategies: choose the *correct* direction, by which we mean lean left if $q_j < q_{-j}$, right if $q_j > q_{-j}$ or lean in the direction j if $q_j = q_{-j}$, or do the opposite. Suppose without loss of generality that $\text{var}[P_\ell] \leq \text{var}[P_r]$ and that the row player corresponds to the player with the smaller value of q_j or the player R if both are the same. We claim that the payoff matrix of this game has the form

	correct	incorrect
correct	$> 0, > 0$	$1, 0$
incorrect	$0, 1$	$< 1, ?$

Observe that the result follows immediately since the game is dominance solvable: it is strictly dominant for row to play *correct* and hence column must also play *correct*.

We now establish the payoffs cell by cell.

correct, correct. This result in two platforms with $q_r \geq q_\ell$. Hence by Proposition 5 neither politician has any cost, and each has a positive probability of winning.

correct, incorrect and *incorrect, correct.* In this case the correct politician receives grassroots support and the incorrect one does not, so the payoffs again follow from Proposition 5.

incorrect, incorrect. There are both left and right platforms, and several cases depending on the comparison between $q_r + \mathrm{E}[P_r]$ and $q_\ell + \mathrm{E}[P_\ell]$. From Proposition 2 :

If $q_r + \mathrm{E}[P_r] < q_\ell + \mathrm{E}[P_\ell]$ neither platform receives grassroots support. From Proposition 4 it follows that both platforms have a positive chance of winning, so neither wins with probability one.

If $q_r + \mathrm{E}[P_r] = q_\ell + \mathrm{E}[P_\ell]$ since $\mathrm{var}[P_\ell] < \mathrm{var}[P_r]$ platform ℓ is supported by grassroots ℓ and platform r receives no grassroots support. Hence r, who is the row player, loses with probability one.

Otherwise platform k is supported by grassroots k. From Proposition 4 it follows that both platforms have a positive chance of winning, so neither wins with probability one. □

4.6. Target Platform Choice

Recall that there is a unique $0 < z^* < 1$ such that $H_\ell(z^*) = H_r(1 - z^*)$. Define the rightwing advantage

$$\rho \equiv \frac{\mathrm{E}[P_r] + \mathrm{E}[P_\ell]}{2} + \frac{\mathrm{var}[P_\ell] - \mathrm{var}[P_r]}{\mathrm{E}[P_r] + \mathrm{E}[P_\ell]}.$$

Since G is continuous and strictly increasing, there is a unique q^* satisfying $G(q^* + \rho) = z^*$.

Proposition 7. *There is a unique Nash equilibrium of the target choice reduced game, it is in pure strategies, and* $q_j = q_{-j} = q^*$.

Proof. As by Proposition 5 the equilibrium will be peaceful, politician expected utility is $p_k B_k$ so this is in fact (normalizing payoffs dividing by B_k) a zero-sum game in the first period. Specifically, Nash equilibrium is given by maximizing the probability of winning p_k. Moreover, by Proposition 1, λ_k, λ_{-k} are exactly determined by q_k, q_{-k}, and by Proposition 3 the advantaged party has $\lambda_{-d} \geq \lambda_d$, $L = \lambda_d V$, with winning probability

$$p_d = \lambda_d/\lambda_{-d} - (1/2)\left(\lambda_d/\lambda_{-d}\right)^2.$$

Each politician by choosing $q_k = q^*$ can guarantee $\lambda_k/\lambda_{-k} \geq 1$ implying that $p_k \geq 1/2$. This proves, despite the lack of continuity, that the minimax Theorem holds. Moreover, if the opponent chooses any other strategy than $q_{-k} = q^*$ then $p_k > 1/2$. Hence the zero sum game has a unique equilibrium in which each politician chooses $q_k = q^*$. □

4.7. Politician Effort Provision Revisited

We now know from Proposition 7 that $q_k = q_{-k} = q^*$ and so $\lambda_k = \lambda_{-k} = 1$. We now reexamine what happens in the politician effort provision subgame.

Proposition 8. *With $\lambda_k = \lambda_{-k} = 1$ in the politician effort provision reduced game a peaceful equilibrium exists if and only if $V \geq \max B_k/2$. Furthermore if $V > \max B_k$ it is the only equilibrium. If $V < \min\{\min B_k, \max B_k/2\}$ then there is no pure strategy equilibrium. In any equilibrium $u_k(G_k, G_{-k}) \geq B_k - B_{-k} - V$. If $B_{-k} \leq B_k$ then in any contested equilibrium $u_{-k}(G_{-k}, G_k) < B_{-k}/2$.*

Notice that if polarization is low the equilibrium must be in mixed strategies: the existence of such an equilibrium is not in question as it follows from the Glicksberg fixed point theorem. While little is known about the structure of such equilibria, what matters is that Proposition 8 establishes a key fact: any contested equilibrium must be less advantageous to the less office motivated politician than a peaceful equilibrium. This is key since, as the first part of Proposition 8 establishes, by creating enough polarization in the first stage a peaceful equilibrium in the second stage can be guaranteed.

Proof. By Proposition 3 the objective function is given by

$$u_k(e_k, e_{-k}) = B_k p(e_k, e_{-k}) - e_k = (1/2)B_k\left(1 + \operatorname{sign}(e_k - e_{-k})\left(\frac{e_{-k} - e_k}{V}\right)^2\right) - e_k.$$

A peaceful equilibrium exists if and only if when $-k$ provides no effort $e_{-k} = 0$ it is optimal for k also to provide no effort. In this case the objective function is convex in e_k so the optimum is either to provide no effort and get $B_k/2$ or provide V units of effort and get $B_k - V$. It follows that there is a peaceful equilibrium $e_k = e_{-k} = 0$ if and only if $V \geq \max B_j/2$, $j \in \{k, -k\}$.

The uniqueness of peaceful equilibrium is given in Proposition 4.

Turning to the existence of contested pure strategy equilibria, we observe that if both politicians provide the same level of effort $\partial u_k/\partial e_k = -1$ so this is an equilibrium only if it is peaceful. If both provide different levels of effort then the one k providing higher effort is on the convex part of the utility function so must provide effort $e_{-k} + AV$ and win for sure. This implies that $-k$ loses for sure so it must be that $e_{-k} = 0$. If this is to be optimal for $-k$ it must be that

$$\frac{\partial u_{-k}(0, V)}{\partial e_{-k}} = \frac{B_{-k}}{V} - 1 \leq 0,$$

that is $V \geq B_j$ for one of the $j \in \{k, -k\}$. We turn to utility. Since $-k$ will never provide more than B_{-k} units of effort k can win for certain by providing an effort of $B_{-k} + V$ yielding an expected utility of $B_k - B_{-k} - V$. Finally, suppose that $B_k > B_{-k}$ and let $c_k(e_k) = e_k/B_k$ denote the linear cost of exerting effort relative to the value of the prize B_k and $c_k(G_k)$ the expected cost where the expectation is taken with respect to the equilibrium distribution of effort choice G_k. From optimality of G_k and symmetry we have

$$p(G_k, G_{-k}) - c_k(G_k) \geq p(G_{-k}, G_{-k}) - c_k(G_{-k}) = 1/2 - c_k(G_{-k}).$$

By subtraction we have

$$p(G_k, G_{-k}) - 1/2 \geq c_k(G_k) - c_k(G_{-k}). \tag{4.1}$$

Reversing the role of the two politicians we also have

$$p(G_{-k}, G_k) - 1/2 \geq c_{-k}(G_{-k}) - c_{-k}(G_k)$$

or since one politician's chance of winning is the other's chance of losing

$$p(G_k, G_{-k}) - 1/2 \leq c_{-k}(G_k) - c_{-k}(G_{-k}).$$

Together with 4.1 this gives

$$c_{-k}(G_k) - c_{-k}(G_{-k}) \geq c_k(G_k) - c_k(G_{-k}) = (B_{-k}/B_k)\left(c_{-k}(G_k) - c_{-k}(G_{-k})\right).$$

Since $B_{-k}/B_k < 1$ it follows that $c_{-k}(G_k) - c_{-k}(G_{-k}) \geq 0$ hence it must also be the case that $c_k(G_k) - c_k(G_{-k}) \geq 0$. From equation 4.1 then $p(G_k, G_{-k}) \geq 1/2$. If $p(G_k, G_{-k}) > 1/2$ then certainly $u_{-k}(G_{-k}, G_k) < B_{-k}/2$. Suppose instead that $p(G_k, G_{-k}) = 1/2$. This implies that if one politician provides zero effort for certain both do so, so that the equilibrium would be peaceful. Hence both provide positive effort with positive probability so both get less than $B_j/2$, $j \in \{k, -k\}$. □

4.8. Main Theorem

We can now prove the main theorem. Recall that $B_w \geq B_{-w}$. We know from Proposition 4 we should have a peaceful equilibrium, so there must be enough polarization for this to be true. The key new idea here is to find a peaceful equilibrium. We do so by observing that if the politician w with greater office motivation chooses not to polarize, that is, $x_w = 0$ then the politician with less office motivation who stands to lose in a contested election can block w from a contested election by providing enough polarization, so this will be an equilibrium. Here are the details of the proof:

Proof. From Proposition 4 the equilibrium is peaceful, and from Proposition 8 both parties have an equal chance of winning giving the politicians expected utility $B_k/2$ and with probability one $V \geq B_w/2$. Grassroots welfare is also given in that result. Second, we show that the lower bound on polarization is achieved by showing that indeed $x_w = 0$ and $x_{-w} = B_w/2$ can be supported as an equilibrium. To do this, we assign a peaceful equilibrium whenever one exists. We apply Proposition 8 to see that for any level of polarization equal to or greater than the equilibrium level there is a peaceful equilibrium in the second stage. In particular on the equilibrium path politician k gets $B_k/2$. Since deviations by w only increase polarization no advantage is derived. Moreover, $-w$ gets $B_{-w}/2$ and by Proposition 8 there is no equilibrium of the politician effort reduced game which gives greater expected utility than this. Hence this constitutes a subgame perfect equilibrium. Third, polarization can be equal to $B_w/2$ with probability one if and only if the politicians employ pure strategies in the second stage. Finally, w can choose a platform of $x_w = 0$ and by Proposition 8 receive at least $B_w - B_{-w} - x_{-w}$ in the third stage. As this must be less than or equal to the equilibrium expected utility of $B_w/2$ we see that $x_{-w} \geq B_w/2 - B_{-w}$. In a least polarized equilibrium $x_{-w} = B_w/2 - x_w$. Substituting for x_{-w} in the inequality we have $B_w/2 - x_w \geq B_w/2 - B_{-w}$ giving the stated upper bound on x_w. Finally, $x_w \geq 0$ and the equality gives $x_{-w} \leq B_w/2$. □

5. Conclusion

We have examined a model in which platforms have two dimensions: a range of policies that are acceptable for supporters and the effort made to implement those policies. We showed how substitutability between grassroots and politicans efforts lead to polarization: politicians create polarization to motivate voters and avoid providing their own effort. This polarization coexists with equal probability that the two parties win, despite underlying asymmetries, due to a Hotelling like effect over the range of acceptable policies. Moreover, the more office motivated politicians are, the more they will exploit polarization. Since the consequences on voters depend on the most office motivated politician, our model suggests that high variance in potential candidates can be pernicious as much as one bad apple can spoil the entire barrel. This is especially true in good times when a great deal of polarization is needed to motivate voters.

What narrative can we tell? Hall (2019) and particularly McCarthy, Poole and Rosenthal (2006) cite evidence that polarization fell during the twentieth century up until about 1980 then started rising.[18] Our view is that good times

[18] Hall (2019) provides by way of explanation the idea that it is far more costly to run for office, so only extremists win. Without rejecting this, we observe that this does not explain why parties alternate in power and why voters should be more polarized.

bring out more office oriented politicians while bad times bring out politicians more interested in solving problems. In the U.S. for example the earlier part of the century was fraught with depression, war, and then cold war. After 1980 the economy boomed, war and cold war were on the way out, and indeed the entire world started experiencing a wave of prosperity unprecedented in history. It makes sense then that in the face of reduced risk and increased prosperity, candidates were interested in holding office primarily out of self interest. In our theory this leads to an increase in polarization. What this means for the future is complicated. In our theory only the most office oriented politician matters: at the current time that is Donald Trump who is exceptionally office oriented by historical standards, and whose rise has, as the theory indicates, coincided with extreme polarization. On the other hand, after the election of Trump, a series of crises began first with Covid, and now with war in Ukraine. We expect, then, that as interest in Donald Trump fades, a more pragmatic and less office oriented group of politicians will emerge and polarization will again begin to decline. In the U.K., where these crises have been exasperated by the Brexit shock, this may have already begun with Sunak and Starmer appearing to be considerably less strident than their predecessors.

References

Aragones, Enriqueta and Thomas R. Palfrey (2002): "Mixed equilibrium in a Downsian model with a favored candidate," *Journal of Economic Theory* 103: 131-161.

Ashworth, Scott and Ethan Bueno De Mesquita (2009): "Elections with platform and valence competition," *Games and Economic Behavior* 67: 191-216.

Baye, Michael R., Dan Kovenock and Caspar G. de Vries (1996): "The all-pay auction with complete information," *Economic Theory* 8: 291–305.

Callander, S. (2008): "Political motivations," *Review of Economic Studies* 75: 671-697.

Carillo, Juan D. and Micael Castanheria (2008): "Information and strategic political polarization," *Economic Journal* 118: 845-874.

Enos, Ryan D., Fowler Anthony and Lynn Vavreck (2013): "Increasing inequality: The effect of GOTV mobilization on the composition of the electorate," *Journal of Politics* 76: 273-288.

Kittsteiner, Thomas and Erik Eyster (2007): "Party platforms in electoral competition with heterogeneous constituencies," *Theoretical Economics* 2: 41-70.

Groseclose, Tim (2001): "A model of candidate location when one candidate has a valence advantage," *American Journal of Political Science*: 862-886.

Green, Donald P. and Alan S. Gerber (2019): *Get Out the Vote: How to Increase Voter Turnout,* Brookings Institution Press.

Hall, A. B. (2019): "Who wants to run?: How the devaluing of political office drives polarization," University of Chicago Press.

Herrera, Helios, David K. Levine and Cesar Martinelli (2008): "Policy platforms, campaign spending and voter participation," *Journal of Public Economics* 92: 501-513.

Hillman, Arye L. and John G. Riley (2006): "Politically contestable rents and transfers," *Economics and Politics* 1: 17-39.

Hirsch, Alexander V. (2022): "Polarization and Campaign Spending in Elections," *Journal of Politics*.

Iaryczower, Matias and Andrea Mattozzi (2013): "On the nature of competition in alternative electoral systems," *Journal of Politics* 75: 743-756.

Kartik, Navin and R. Preston McAfee (2007): "Signaling character in electoral competition," *American Economic Review* 97: 852-870.

Levine, David K. and Cesar Martinelli (2022): "Razor Thin Elections," mimeo EUI

Levine, David K. and Andrea Mattozzi (2020): "Voter turnout with peer punishment," *American Economic Review* 110: 3298-3314.

McCarty, N., K. T. Poole and H. Rosenthal (2006): *Polarized America: The Dance of Ideology and Unequal Riches*, MIT Press.

Meirowitz, Adam (2007): "Probabilistic voting and accountability in elections with uncertain policy constraints," *Journal of Public Economic Theory* 9: 41-68.

Schofield, Norman and Gary Miller (2007): "Elections and activist coalitions in the United States," *American Journal of Political Science* 51: 518-531.

Schofield, Norman (2006) "Equilibria in the spatial stochastic model of voting with party activists," *Review of Economic Design*,10(: 183–203.

Stokes, David E. (1963): "Spatial models of party competition," *American Political Science Review* 57: 368-377.

Venkatesh, Raghul S. (2020): "Political activism and polarization," *Journal of Public Economic Theory* 22: 1530-1558.