are three cases: that in which $h(t_j) < \frac{1}{2}$ (firm j gets less than ha... if it is the first to release its product), that in which $h(t_j) = \frac{1}{2}$, and that in which $h(t_j) > \frac{1}{2}$.)

? EXERCISE 80.2 (A fight) Each of two people has one unit of a resource. Each person chooses how much of the resource to use in fighting the other individual and how much to use productively. If each person i devotes y_i to fighting, then the total output is $f(y_1, y_2) \geq 0$ and person i obtains the fraction $p_i(y_1, y_2)$ of the output, where

some value between 0-1

$$p_i(y_1, y_2) = \begin{cases} 1 & \text{if } y_i > y_j \\ \frac{1}{2} & \text{if } y_i = y_j \\ 0 & \text{if } y_i < y_j. \end{cases}$$

The function f is continuous (small changes in y_1 and y_2 cause small changes in $f(y_1, y_2)$), is decreasing in both y_1 and y_2 (the more each player devotes to fighting, the less output is produced), and satisfies $f(1, 1) = 0$ (if each player devotes all her resource to fighting, then no output is produced). (If you prefer to deal with a specific function f, take $f(y_1, y_2) = 2 - y_1 - y_2$.) Each person cares only about the amount of output she receives, and prefers to receive as much as possible. Specify this situation as a strategic game and find its Nash equilibrium (equilibria?). (Use a direct argument: first consider pairs (y_1, y_2) with $y_1 \neq y_2$, then those with $y_1 = y_2 < 1$, then those with $y_1 = y_2 = 1$.)

I've specified the players and actions. But I'm confused about the payoffs. Is the utility function defined by

$$u_i = \begin{cases} 1(2 - y_i - y_j) & if\ y_i > y_j \\ \frac{1}{2}(2 - y_i - y_j) & if\ y_i = y_j \\ 0(2 - y_i - y_j) & if\ y_i < y_j \end{cases}$$

or

$$u_i = \begin{cases} 1(2 - y_i - y_j) - y_i & if\ y_i > y_j \\ \frac{1}{2}(2 - y_i - y_j) - y_i & if\ y_i = y_j \\ 0(2 - y_i - y_j) - y_i & if\ y_i < y_j \end{cases}$$

If the second function is correct and the first one is wrong, can you explain why it makes sense to have negative utility where player i *loses* the fight so that u = -y_i ? If each player started off with 1 unit of resource to begin with (and so started with eg. u=1) and they spent y_i on the fight and gave the rest (1- y_i) to the other player because they lost, isn't utility equal to 0 not - y_i?

Then, could anyone be so kind as to explain how to go about drawing the best responses of each player? Is there any easier way of thinking about this problem?

Thank you